Multivalent Uniformly Convex Functions by Using Differential Operator

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This section is devoted to regular and multi-valent mapping by using differential Op_{tor} in the $\mathcal{U}_{\mathcal{D}}$. We study various exciting things for this novel class prior to multivalent mappings.

Allow S exist the class of the mappings

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n, \quad a^n \geq 0 \text{ (p} \in N) \tag{1.1}$$

whichever regular & p- valent within effective $\mathcal{U}_{\mathcal{D}}$, U = {z: |z| < 1}

Furthermore S^* be effective sub cl_{ss} prior to S containing to mappings

$$f(z) = z^p - \sum_{n=p+1}^{\infty} a_n z^n, a^n \geq 0 \text{ (p} \in N) \tag{1.2}$$

$$\text{Real}\left\{1 + \frac{zf''(z)}{f'(z)} - \alpha\right\} > \beta\left|\frac{zf'(z)}{f(z)} - 1\right|, \quad (z \in \mathcal{U}) \tag{1.3}$$

Wherever $-1\alpha \leq 1, \quad \beta \geq 0$ & p∈ N

(ii) A mappings f(z)∈ S suppose to subsist within effective cl_{ss} $\mathcal{UCV}$ (α, β) to consistently β-$\mathcal{CV}$ & satisfy

$$\text{Real}\left\{1 + \frac{zf''(z)}{f'(z)} - \alpha\right\} > \beta\left|\frac{zf''(z)}{f'(z)} - 1\right|, \quad (z \in \mathcal{u}) \tag{1.4}$$

wherever $\alpha \leq 1, \quad \beta > 0$ and p∈ N

From above (1.3) & (1.4)

f(z)∈ $\mathcal{UCN}(\alpha, \beta)$ do comparable toward $zf'(z) \in S_p(\alpha, \beta)$ (1.5)

Hd_{pro} of f(z) ,g(z)∈ S can be define as

$$f * g(z) = z^p + \sum_{k=p+1}^{\infty} a_n b_n z^n \quad (z \in \mathcal{u}), \text{p} \in N \tag{1.6}$$

Concerning effective mapping f(z)∈ S ,without help classify affecting subsequent

$$I^0 f(z) = f(z), \quad I^1 f(z) = zf'(z) + \frac{1+p}{z^p}$$

along with k = 2,3,4,……..

$$= z^p + \sum_{n=p+1}^{\infty} n(k) a_n z^n,, \quad \text{p} \in N \tag{1.7}$$

Somewhere I^k is the same as diff. Op_{tor} , Ghanim & Darus [2], S.K.Lee, S. Khairnar with S. Rajas [9] have studied this Op_{tor} widely.

Let $S^*(\alpha, \beta)$ ∈S consisting of the mapping of the form (1.1) and satisfy

$$\left|\frac{\dfrac{z\left(I^k f(z)\right)' - p}{I^k f(z)}}{\dfrac{\beta z\left(I^k f(z)\right)'}{I^k f(z)} - \alpha pz\dfrac{\left(I^k f(z)\right)'}{I^k f(z)}}\right| < \mu \tag{1.8}$$

where $-1 \leq \alpha < \beta \leq 1$ and $0 < \mu \leq 1 \left(z \in \mathcal{u}\right)$.

Also let $S^{**}(\alpha,\beta) = S^*(\alpha, \beta) \cap S^*$

3.2.1Coefficient Estimate

Here we obtained a essential & enough situation for function f(z) inside effective cl_{ss} $S^*(\alpha, \beta) and S^{**}(\alpha,\beta)$.

Theorem **1:** A mapping of the equation (1.1) is in $S^*(\alpha,\beta)$ iff

$$\sum_{n=1}^{\infty}\left[(n-p)+\mu n(\beta-\alpha p)\right]n(k)|a_n|<\mu p(\beta-\alpha p), \qquad (1.9)$$

where -1≤ $\alpha<\beta\leq 1$ and 0< $\mu\leq 1$ and p∈ N.

Proof **:** It's enough to illustrate so as to

$$\left|\frac{\dfrac{z\left(I^k f(z)\right)'}{I^k f(z)}-p}{\dfrac{\beta z\left(I^k f(z)\right)'}{I^k f(z)}-\alpha pz\dfrac{\left(I^k f(z)\right)'}{I^k f(z)}}\right|<\mu$$

as f(z)∈ $S^*(\alpha,\beta)$ we have

$$\left|\frac{\dfrac{z\left(I^k f(z)\right)'}{I^k f(z)}-p}{\dfrac{\beta z\left(I^k f(z)\right)'}{I^k f(z)}-\alpha pz\dfrac{\left(I^k f(z)\right)'}{I^k f(z)}}\right|\leq\mu\left(z\in\mathcal{U}\right),\text{p}\in N$$

$$=\left|\frac{\dfrac{pz^p+\sum_{n=p+1}^{\infty}n(k)na_nz^n}{z^p+\sum_{n=p+1}^{\infty}n(k)a_nz^n}-p}{\dfrac{\beta pz^p+\beta\sum_{n=p+1}^{\infty}n(k)na_nz^n}{z^p+\sum_{n=p+1}^{\infty}n(k)a_nz^n}-\alpha p\dfrac{pz^p+\sum_{n=p+1}^{\infty}n(k)na_nz^n}{z^p+\sum_{n=p+1}^{\infty}n(k)a_nz^n}}\right|\leq\mu$$

$$=\left|\frac{pz^p+\sum_{n=p+1}^{\infty}n(k)na_nz^n-pz^p-\sum_{n=p+1}^{\infty}n(k)a_nz^n}{pz^p(\beta-\alpha p)+\sum_{n=p+1}^{\infty}(\beta-\alpha p)n(k)na_nz^n}\right|\leq\mu$$

$$\sum_{n=1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|a_n||z^n|<\mu p(\beta-\alpha p)\;|z^p|$$

Allowing the value of z→ -1 taking place effective $\mathcal{R}e_{al}Ax_{is}$, without help obtained

$$\sum_{n=p+1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|a_n|<\mu p(\beta-\alpha p)$$

Theorem **2:** A essential and enough stipulation in favor of f(z) prior to the structure (1.2) toward exist effective $c\ell_{ss}$ S^{**}(α,β).

$$\sum_{n=p+1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|a_n|\leq\mu p(\beta-\alpha p) \qquad (1.10)$$

where -1≤ $\alpha\leq\beta$ and 0< $\mu\leq 1$ & p∈ N.

Proof **:** It's enough to illustrate so as to

$$\left|\frac{\frac{z\left(I^k f(z)\right)'-p}{I^k f(z)}}{\frac{\beta z\left(I^k f(z)\right)'}{I^k f(z)}-\alpha p\frac{z\left(I^k f(z)\right)'}{I^k f(z)}}\right| \le \mu$$

We enclose

$$\left|\frac{\frac{pz^p+\sum_{n=p+1}^{\infty} n(k)na_n z^n}{z^p+\sum_{n=p+1}^{\infty} n(k)a_n z^n}-p}{\frac{\beta pz^p+\beta\sum_{n=p+1}^{\infty} n(k)na_n z^n}{z^p+\sum_{n=p+1}^{\infty} n(k)a_n z^n}-\alpha p\frac{pz^p+\sum_{n=p+1}^{\infty} n(k)na_n z^n}{z^p+\sum_{n=p+1}^{\infty} n(k)a_n z^n}}\right| \le \mu$$

$$\sum_{n=1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|a_n||z^n| \le \mu p(\beta-\alpha p)\,|z^n|$$

Allowing the value of z→ − 1 with effective $\mathcal{R}e_{al}Ax_{is}$, without help acquire

$$\sum_{n=p+1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|a_n| < \mu p(\beta-\alpha p)$$

The S^{**}(α,β) remain closed underneath linear combination we will prove this in the following theorem.

Theorem **3:** If f(z) is definite through (1.2) and

$$g(z)=z^p-\sum_{n=p+1}^{\infty} b_n z^n$$

live in the class S^{**}(α,β).Then the function

$$h(z)=(1-\epsilon)f(z)+\epsilon g(z)=z^p-\sum_{n=p+1}^{\infty}\eta_n z^n \qquad (1.11)$$

Is as well within S^{**}(α,β) wherever

$\eta_n=(1-\epsilon)a_n+\epsilon b_n$ $\quad 0\le \epsilon \le 1.$

Proof **:** As the mappings f(z) & g(z) hold inside S^{**}(α,β), so we include

$$\sum_{n=p+1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|a_n| < \mu p(\beta-\alpha p)$$

And

$$\sum_{n=p+1}^{\infty}[(n-p)+\mu n(\beta-\alpha p)]n(k)|b_n| < \mu p(\beta-\alpha p)$$

Then

h(Z) = $(1-\epsilon)f(z)+\epsilon g(z)$

$$=(1-\epsilon)z-\sum_{n=p+1}^{\infty}a_n z^n+\epsilon\left(z-\sum_{n=p+1}^{\infty}b_n z^n\right)$$

$$=z^p-\sum_{n=p+1}^{\infty}\left|\left[(1-\epsilon)a_n+\epsilon b_n\right]\right|z^p$$

$$= z^p - \sum_{n=p+1}^{\infty} c_n z^n$$

when $c_n = (1-\epsilon)a_n + \epsilon b_n$

Now consider

$$\sum_{n=p+1}^{\infty} [(n-p) + \mu n(\beta - \alpha p)]n(k)|c_n|$$

$$= \sum_{n=p+1}^{\infty} [(n-p) + \mu n(\beta - \alpha p)]n(k)|(1-\epsilon)a_n + \epsilon b_n|$$

$$\leq (1-\epsilon) \sum_{n=p+1}^{\infty} \left[(n-p) + \mu n(\beta - \alpha p)\right] n(k)|a_n|$$

$$+\epsilon \sum_{n=p+1}^{\infty} \left[(n-p) + \mu n(\beta - \alpha p)\right] n(k)|a_n|$$

$\leq (1-\epsilon)\mu p(\beta - \alpha p) + \epsilon\mu p(\beta - \alpha p)$

$=\mu p(\beta - \alpha p)$

Thus we get

$$\sum_{n=p+1}^{\infty} [(n-p) + \mu n(\beta - \alpha p)]n(k)|a_n| < \mu p(\beta - \alpha p)$$

Hence h(z)∈ S^{**}(α,β)

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