Application of Special Function in Solving Partial Differential Equation

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Abstract

Partial differential equations (PDEs) form the mathematical foundation for modeling a wide spectrum of phenomena in physics, engineering, and applied sciences, including heat transfer, wave propagation, fluid dynamics, and quantum mechanics. Despite their broad applicability, obtaining closed-form solutions to PDEs remains a significant challenge, particularly for problems defined on complex domains or involving non-trivial boundary and initial conditions. In this context, special functions serve as indispensable analytical tools that enable the systematic construction of exact and approximate solutions. This work investigates the role of special functions—such as Bessel functions, Legendre polynomials, Hermite functions, and Laguerre polynomials—in the analytical treatment of classical and modern PDEs. Using the method of separation of variables, complex PDEs are decomposed into ordinary differential equations whose solutions naturally lead to families of orthogonal special functions. These functions form complete bases, allowing the representation of solutions as convergent series expansions. The study focuses on canonical equations, including the heat equation, wave equation, and Laplace equation, analyzed in Cartesian, cylindrical, and spherical coordinate systems. Particular attention is given to boundary value problems where eigenvalue formulations arise, leading to Sturm–Liouville systems and orthogonality conditions that justify the use of special functions. The convergence properties, normalization, and completeness of these functions are also discussed to ensure the validity and applicability of the resulting solution.