

Mathematical Properties of Special Functions and Their Applications

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Abstract

Special functions constitute an important class of mathematical functions that arise naturally in differential equations, mathematical physics, engineering, probability, and applied sciences. Functions such as Gamma, Beta, Bessel, Legendre, Hermite, Laguerre, and hypergeometric functions provide analytical solutions to a wide range of mathematical and physical problems. This paper presents a study of the fundamental mathematical properties of selected special functions, with particular emphasis on their recurrence relations, integral representations, generating functions, differential equations, orthogonality properties, and asymptotic behaviour. The relationships between different families of special functions are also discussed to demonstrate their structural and analytical significance. Particular attention is given to the role of special functions in obtaining exact and approximate solutions to differential and integral equations. Their applications in mathematical modelling, quantum mechanics, heat and wave propagation, signal processing, probability theory, and engineering are briefly examined. The paper also highlights the importance of computational techniques and symbolic mathematical software in evaluating special functions and investigating their properties for complex problems. Understanding these functions provides a systematic framework for analysing mathematical models that cannot be expressed adequately through elementary functions. The study emphasizes that continued development of analytical and computational methods for special functions can contribute to more efficient solutions of advanced problems in mathematics, physics, and engineering. The paper therefore provides a conceptual overview of their mathematical significance and interdisciplinary applications.

Keywords: Special Functions, Gamma Function, Beta Function, Bessel Functions, Orthogonal Functions, Differential Equations, Mathematical Modelling, Applied Mathematics