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Review of Literature Nondifferentiable Multi Objective

Ratnappa, Research Scholar, Department of Mathematics, SunRise University, Alwar, Rajasthan (India)
Dr. Satendra Kumar, Professor, Department of Mathematics, SunRise University, Alwar, Rajasthan (India)
Email: ratnappah@gmail.com

Abstract

Multiobjective optimization problems have been applied in various fields of science, where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Researchers study multiobjective optimization problems from different viewpoints and, then there exist different goals when setting and solving them. The goal may be finding a set of Pareto optimal solutions, and/or qualifying the trade-offs in satisfying the different objectives, and/or finding a single solution that satisfies the preferences of a human decision making. Motivated with these observations, there has been an increasing interest in studying optimality and duality for nondifferentiable multiobjective programming problems.

Keywords: Characteristic, Mathematical, Nondifferentiable Multi Objective

Introduction: A number of researchers have discussed optimality and duality for a class of nondifferentiable problem containing the square root of a positive semi-definite quadratic form. Mond (1974) presented Wolfe type duality while Chandra et al. (1985) investigated Mond-Weir type duality for this class of problems. Later, Zhang and Mond (1997) validated various duality results for the problem under generalized invexity conditions, it is observed that the popularity of this kind of problems seems to originate from the fact that, even though the objective functions, and/or constraint function are non-smooth, a simple and elegant representation for the dual to this type of problems may be obtained. Obviously non-smooth mathematical programming duals with more general type functions by means of generalized sub differentials. However, the square root of a positive semidefinite quadratic form is one of a few cases of nondifferentiable functions for which sub gradient can explicitly be written.

Duality mathematical programming is used in Economics, Control Theory, Business and other diverse fields. In mathematical programming, a pair of primal and dual problems are said to be symmetric when the dual of the dual is the primal problem, i.e., when the dual problem is expressed in the form of the primal problem, then it does happen that its dual is the primal problem. This type of dual problem was introduced by Dorn [1], later on Mond and Weir [2] studying them under weaker convexity assumptions.

Antczak [3] introduced the notion of G-invex function obtaining some optimality conditions which he himself [4] comprehends to be a Gf-invex function, deriving optimality conditions for a multiobjective nonlinear programming problem. Ferrara and Stefaneseu [5] also discussed the conditions of optimality and duality for multiobjective programming problem, and Chen [6] considered multiobjective fractional problems and its duality theorems under higher-order (F,α,ρ,d)- convexity.

In recent years, several definitions such as nonsmooth univex, nonsmooth quasiunivex, and nonsmooth pseudoinvex functions have been introduced by Xianjun [7]. By introducing these new concepts, sufficient optimality conditions for a nonsmooth multiobjective problem were obtained and, a fortiori, weak and strong duality results were established for a Mond-Weir type multiobjective dual program.

Jiao [8] introduced new concepts of nonsmooth $K-\alpha-dI$-invex and generalized type I univex functions over cones by using Clarke's generalized directional derivative and dI-invexity for a nonsmooth vector optimization problem with cone constraints. Op. cit. also established sufficient optimality conditions and Mond-Weir type duality results under $K-\alpha-dI$-invexity and type I cone-univexity assumptions. Very recently Dubey et al. [9] studied further Mond-Weir type dual model multiobjective programming problems over arbitrary cones.

Pitea and Postolache [10] developed the study of a new class of multi-time multiobjective variational problems of minimizing a vector of functionals of curvilinear integral type by means of which they were able to obtain results concerning duals of Mond-

Weir type, generalized Mond-Weir-Zalmai type and under some assumptions of (ρ,b)-quasi-invexity, proving that the value of the objective function of the primal cannot exceed the value of the dual. And Pitea and Antczak [11] provided additional duality Mond-Weir type results and in the sense of Wolfe for multi-time multiobjective variational problems with univex functionals.

Review of literature:

A realistic solution concept associated with a multiobjective optimization problem is that named Pareto (or efficient) solution, which is more difficult to be treated from a mathematical point of view than the notion of weak Pareto (or weakly efficient) solution. This work provides a complete description of the efficient solution set, when the objective functions are defined on the real line. This is motivated, besides theoretical aspects, also by a numerical point of view, since most algorithms in scalar minimization involve the solvability of a one-dimensional optimization problem to find the next iterate. It is expected that the same situation occurs in the multiobjective optimization problem. We first consider the case when all the objective functions are semistrictly quasiconvex, and afterwards we consider the same problem under quasiconvexity along with some additional assumptions. The latter allows us to deal with the general bicriteria optimization problem under quasiconvexity. Several examples showing the applicability of our results are presented, and an algorithm is proposed to compute the whole efficient solution set(1,14,19).

Convexity plays a vital role in many aspects of mathematical programming including optimality conditions and duality theorems, see for example Mangasarian [11] and Bazaraa et al. [3]. To relax convexity assumptions imposed on the functions in theorems on optimality and duality, various generalized convexity concepts have been proposed. Hanson [7] introduced the class of invex functions. Later, Hanson and Mond [8] defined two new classes of functions called type-I and type-II functions, and sufficient optimality conditions were established by using the concepts. Rueda and Hanson [19] further extended type-I functions to the classes of pseudo-type-I and quasi-type-I functions and obtained sufficient optimality conditions for a nonlinear programming problem involving these classes of functions. Kaul et al. [10] considered a multiple objective nonlinear programming problem involving generalized type-I functions and obtained some results on optimality and duality, where the Wolfe and Mond–Weir duals are considered. Univex functions were introduced and studied by Bector et al. [4]. Rueda et al. [20] obtained optimality and duality results for several mathematical programs by combining the concepts of type-I and univex functions. Mishra [14] considered a multiple objective nonlinear programming problem and obtained a few results on optimality, duality and saddle point of a vector valued Lagrangian by combining the concepts of type-I, pseudo-type-I, quasi-type-I, quasi-pseudo-type-I, pseudo-quasitype-I and univex functions. Aghezzaf and Hachimi [1] introduced new classes of weak strictly pseudoinvex, strong pseudoinvex, weak quasi invex, weak pseudoinvex and strong quasi invex functions. It is known that, despite substituting invexity for convexity, many theoretical problems in differentiable programming can be solved [6,7,9]. But the corresponding conclusions cannot be obtained in nondifferentiable programming with the aid of invexity introduced by Hanson [7] because the existence of a derivative is required in the definition of invexity. There exists a generalization of invexity to locally Lipschitz functions, with derivative replaced by the Clarke generalized gradient [5,12,13,15, 16,18]. However, Antczak [2] used directional derivative, in association with a hypothesis of an invex kind following Ye [23]. The necessary optimality conditions in Antczak [2] are different from those cited in the literature. In the present paper, we consider a nondifferentiable and multiobjective programming problem. A few Karush–Kuhn–Tucker type of sufficient optimality conditions are derived for a (weakly) Pareto efficient solution to the problem involving the new classes of directionally differentiable generalized d-univex functions by combining the concepts of univex functions in Bector et al. [4], weak strictly pseudoinvex, strong pseudoinvex, weak quasi invex, weak pseudoinvex and strong quasi invex functions in Aghezzaf and Hachimi

[1] and d-invex functions in Antczak [2]. Furthermore, the Mond–Weir type and general Mond– Weir type of duality results are also obtained in terms of right differentials of the aforesaid functions involved in the multiobjective programming problem. The results in this paper extend many earlier works in the literature.
The study of optimality conditions and duality for (weakly) efficient solutions in fractional multiobjective/vector optimization problems has been made intensively by many researchers; see e.g., [1], [4], [12], [13], [17], [19] and the references therein. One of the most commonly used approaches for examining a fractional multiobjective optimization problem is to employ the separation theorem of *convex sets* (see e.g., [18]) to provide necessary conditions for (weakly) efficient solutions of the considered problem and to exploit various kinds of (generalized) convex/or invex functions to formulate sufficient conditions for such solutions. It is interesting to observe further that since the kinds of (generalized) invex functions mentioned above have been constructed via the convexified/Clarke subdifferential of locally Lipschitz functions, we need to use the separation theorem of convex sets in the schemes of proof tacitly.
A remarkable feature of a fractional multiobjective optimization problem is that its objective function is generally *not* a convex function. Even under more restrictive concavity/convexity assumptions fractional multiobjective optimization problems are generally *nonconvex* ones. Meantime, the (approximate) *extremal principle* [16], which plays a key role in variational analysis and generalized differentiation, has been well-recognized as a variational counterpart of the separation theorem for *nonconvex* sets. Hence using the extremal principle and other advanced techniques of variational analysis and generalized differentiation to establish optimality conditions seems to be suitable for *nonconvex* and *nondifferentiable* (nonsmooth) fractional multiobjective optimization problems.
In this work, we apply some advanced tools of variational analysis and generalized differentiation (e.g., the nonsmooth version of Fermat's rule, the limiting/Mordukhovich subdifferential of maximum functions, and the sum rule as well as the quotient rule for the limiting subdifferential [16]) to establish necessary conditions for local (weakly) efficient solutions of a nondifferentiable fractional multiobjective optimization problem involving an *infinite* number of inequality constraints (called also *nondifferentiable fractional semi-infinite multiobjective optimization problem*). We refer the reader to [7] for some results in this direction for problems with *finitely* many inequality and equality constraints. Sufficient conditions for such solutions to the considered problem are also provided by means of introducing (strictly) generalized convex functions defined in terms of the limiting subdifferential for a family of locally Lipschitz functions. Along with optimality conditions, we propose a dual problem to the primal one and examine weak, strong and converse duality relations under assumptions of (strictly) generalized convexity. Moreover, examples are given for analyzing and illustrating the obtained results.
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