International Advance Journal of Engineering, Science and Management (IAJESM)
ISSN -2393-8048, January-June 2014, Submitted in April 2014, iajesm2014@gmail.com

PROBABILITY DISTRIBUTION OF INTEGRAL INVOLVING HYPERGEOMETRIC FUNCTION

Ganesh Choudhary, Research Scholar, Department of Mathematics, Maharja Ganga Singh University, Bikaner (Rajasthan).
Dr. V. K. Gaur, Research Supervisor, Department of Mathematics, Maharja Ganga Singh University, Bikaner (Rajasthan).

4.1 MAIN INTEGRALS

In this section, the following probability distribution of thirty nine integrals involving hypergeometric functions have been obtainedin the form of a single integral.

$$\int_0^1 x^{c-1}(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-i-1}F_1^2(a,1+j-a',e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e+i+1)\Gamma(e-c-\frac{1}{2}(i+|i|))\Gamma(c-\frac{1}{2}(j+|j|))\Gamma(a-\frac{1}{2}(i+j+|i+j|))}{2^{2a-i-j}(1+\alpha)^c(1+\beta)^{c-e+i+1}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma 2c-e-a+i+1)} x\{D_{i,j}\frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1-+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^i}{4})((-1)^i-1+[-j/2])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}+\left[-\frac{j}{2}\right])}+ E_{i,j}\frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+1)\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^j}{4})((1-(-1)^i)+[-\frac{j}{2}+\frac{1}{2}])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}-\frac{1}{2}+\left[-\frac{j}{2}+\frac{1}{2}\right])} \quad \ldots\ldots\ldots(4.1.1)$$

for i,j=0,±1, ±2, ±3.

Also, provided Re(e)>0,Re(c-e+i+1)>0 for i=0, ±1, ±2, ±3 and Re(c)>j for j=1,2,3.also the constants α and β are such that no one of the expressions 1+α,1+β and 1+αx+β(1-x), where 0≤ x ≤1, is zero. Again, as usual [x] is the greatest integer less than or equal to x. the coefficient $D_{i,j}$ and $E_{i,j}$ are given in the tabular form.

So by well known definition of probability distributions we have:

$$F(x)=\frac{\{\frac{\Gamma(e)\Gamma(c-e+i+1)\Gamma\left(e-c-\frac{1}{2}(i+|i|)\right)\Gamma\left(c-\frac{1}{2}(j+|j|)\right)\Gamma\left(a-\frac{1}{2}(i+j+|i+j|)\right)}{2^{2a-i-j}(1+\alpha)^c(1+\beta)^{c-e+i+1}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+i+1)}x \{D_{i,j}\frac{\Gamma\left(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1\mp(-1)^i)\right)}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^i}{4})((-1)^i-1+[-j/2])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}+\left[-\frac{j}{2}\right])}+ E_{i,j}\frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+1)\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^j}{4})((1-(-1)^i)+[-\frac{j}{2}+\frac{1}{2}])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}-\frac{1}{2}+\left[-\frac{j}{2}+\frac{1}{2}\right])}\}}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-i-1}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a,1+j-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

4.2 SPECIAL CASES

1. If we set I,j=0, ±1, ±2, in (4.2.1),we get wenty five integrals obtained earlier by Nagar [108] and gaur(2003).

2. On the other hand, fourteen integrals for different values of I and j other than obtained by Nagar[108] and gaur(2003).

First Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+3}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}F_1^2(a,1-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3))\Gamma(c)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+4)}x[\{-(a+2)(a-3)+3c(c+3)-e(3c-e+5)\}\frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+2)\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}+\{(a+1)(a-2)-c(c+3)-e(c-e+3)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})} \ldots\ldots\ldots(4.2.1)$$

Provided Re(c)>0,Re(c-e+4)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3))\Gamma(c)}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma 2c-e-a+4)}x\,[\{-(a+2)(a-3)+3c(c+3)-e(3c-e+5)\}\frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+2)\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}+\{(a+1)(a-2)-c(c+3)-e(c-e+3)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, 1-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Second Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+3}[.1+\alpha x+\beta(1-x)]^{-2c+e-3}F_1^2(a,-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e+3)\Gamma(e-c-2))\Gamma(c)}{2^{2a+1}(1+\alpha)^{c}(1+\beta)^{c-e+3}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+3)}x[\{-(a+1)(a-2)+c(a+c+3)-e(2c-e+3)\}\frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+\frac{3}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}+\{(a-1)(a+2)+c(a-c+3)-e(2c-e+3)\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)} \quad \text{..........................(4.2.2)}$$

Provided Re(c)>0,Re(c-e+4)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\frac{\Gamma(e)\Gamma(c-e+3)\Gamma(e-c-2))\Gamma(c)}{2^{2a+1}(1+\alpha)^{c}(1+\beta)^{c-e+3}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma 2c-e-a+3)}x\,[\{-(a+1)(a-2)+c(a+c+3)-e(2c-e+3)\}\frac{\Gamma\left(c-\frac{a}{2}-\frac{e}{2}+\frac{3}{2}\right)\Gamma\left(\frac{e}{2}-\frac{a}{2}\right)}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}+\{(a-1)(a+2)+c(a-c+3)-e(2c-e+3)\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-41}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a,-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Third Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-2}F_1^2(a, 1-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e+2)\Gamma(e-c-1))\Gamma(c)}{2^{2a+2}(1+\alpha)^{c}(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+2)}x[\{(a-1)(a+2)-2c(c+2)-e(3c-e+3)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+1)\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{1}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}+\{-a(a+1)-e(c-e+1)\}\frac{\Gamma(c+\frac{3}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}] \quad \text{..........................(4.2.3)}$$

Provided Re(c)>0,Re(c-e+2)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\frac{\Gamma(e)\Gamma(c-e+2)\Gamma(e-c-1))\Gamma(c)}{2^{2a+2}(1+\alpha)^{c}(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+2)}x\,[\{(a-1)(a+2)-2c(c+2)-e(3c-e+3)\}\frac{\Gamma\left(c-\frac{e}{2}-\frac{a}{2}+1\right)\Gamma\left(\frac{e}{2}-\frac{a}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{e}{2}+\frac{a}{2}+\frac{1}{2}\right)-\Gamma\left(c-\frac{e}{2}+\frac{a}{2}+2\right)}+\{-a(a+1)-e(c-e+1)\}\frac{\Gamma\left(c+\frac{3}{2}-\frac{e}{2}-\frac{a}{2}\right)\Gamma\left(\frac{e}{2}-\frac{a}{2}\right)}{\Gamma\left(\frac{e}{2}+\frac{a}{2}+1\right)\Gamma\left(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2}\right)}]}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(a, 1-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Fourth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(a,-2-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx = \frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c)}{2^{2a+3}(1+\alpha)^{c}(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(2c-e-a+1)}x\,[\{e(2c-e+1)+a(a-c+1)\}\frac{\Gamma(c+\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+1)-\Gamma(c-\frac{e}{2}+\frac{1}{2a}+\frac{3}{2})}+\{e(2c-e+1)+(a+2)(a+c+1)\}\frac{\Gamma(c+1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}] \qquad (4.2.4)$$

Provided Re(c)>0,Re(c-e+1)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{[\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c)}{2^{2a+3}(1+\alpha)^{c}(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(2c-e-a+1)}x\,\{e(2c-e+1)+a(a-c+1)\}\frac{\Gamma\left(c+\frac{1}{2}-\frac{e}{2}-\frac{a}{2}\right)\Gamma\left(\frac{e}{2}-\frac{a}{2}\right)}{\Gamma\left(\frac{e}{2}+\frac{a}{2}+1\right)-\Gamma\left(c-\frac{e}{2}+\frac{1}{2a}+\frac{3}{2}\right)}+\{e(2c-e+1)+(a+2)(a+c+1)\}\frac{\Gamma(c+1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}]}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(a,-2-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Fifth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+3}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}F_1^2\left(a,2-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx = \frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-1)}{2^{2a+1}(1+\alpha)^{c}(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}x\ [\{-a(a-1)(a+e-$$

3)+c(a+c)} $\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}$ **+{(a-1)(a-e+1)+c(a-c-2)}** $\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}$**]..........................(4.2.5)**

Provided Re(c)>0,Re(c-e+4)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\left[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-1)}{2^{2a+1}(1+\alpha)^{c}(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}\text{x}\ \{-a(a-1)(a+e-3)+c(a+c)\}\frac{\Gamma\left(c+2-\frac{e}{2}-\frac{a}{2}\right)\Gamma\left(\frac{e}{2}-\frac{a}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{e}{2}+\frac{a}{2}-\frac{1}{2}\right)-\Gamma\left(c-\frac{e}{2}+\frac{a}{2}+1\right)}+\{(a-1)(a-e+1)+c(a-c-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}\right]}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a,-a;\ e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Sixth Formula

$\int_0^1 x^{c-1}(1-x)^{c-e+3}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}F_1^2(a,3-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx =$ $\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-2)}{2^{2a+2}(1+\alpha)^{c}(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}$ **x [{-(a-1)(a-2)+c(2c-e+2)}** $\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}$ **+{(a-1)(a-2)-c(e-2)}** $\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{1}{2})}$**]..........................(4.2.6)**

Provided Re(c)>0,Re(c-e+4)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\left[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-2)}{2^{2a+2}(1+\alpha)^{c}(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}\text{x}\ \{-(a-1)(a-2)+c(2c-e+2)\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}+\{(a-1)(a-2)-c(e-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{1}{2})}\right]}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, 3-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Seventh Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+3}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}F_1^2(a,4-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)} x[\{e(2c-e)-(a-6)(a-c+e)-c-11\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2})}+\{(-e(2c-e+a+2)+(a+3)(a+c+1)-6a\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-2)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{1}{2})}]\text{.........................}(4.2.7)$$

Provided Re(c)>0,Re(c-e+4)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}x\ \{e(2c-e)-(a-6)(a-c+e)-c-11\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2})}+\{(-e(2c-e+a+2)+(a+3)(a+c+1)-6a\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-2)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{1}{2})}]}{\int_0^1 x^{c-1}(1-x)^{c-e+i}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, 4-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Eighth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2(a,4-a,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+1)} x[\{(a+3)(1+a-c)+e(2c-e+1)-6a\}\frac{\Gamma(c+\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-2)\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})}+\{-(a-7)(a+c-2)-e(2c-e+1)-3(a-1)\}\frac{\Gamma(c+1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}]\text{.........................}(4.2.8)$$

Provided Re(c-3)>0,Re(c-e+1)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{[\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+1)}x\ [\{(a+3)(1+a-c)+e(2c-e+1)-6a\}\frac{\Gamma(c+\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-2)\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})}+\{-(a-7)(a+c-2)-e(2c-e+1)-3(a-1)\}\frac{\Gamma(c+1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}]}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-4}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(a, 4-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Ninth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e-1}[1+\alpha x+\beta(1-x)]^{-2c+e} F_1^2\left(a,3-a;\ e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx =$$

$$\frac{\Gamma(e)\Gamma(c-e)\Gamma(c-3)\Gamma(a-2)}{2^{2a-2}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a)}\text{x}\Big[\{(a-1)(a-2)+(e-c)(2c-e-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}+\{(a-1)(a-2)+(e-2)(c-e)\}\frac{\Gamma(c+\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})}\Big]\text{.........................(4.2.9)}$$

Provided Re(c-3)>0,Re(c-e)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\left[\frac{\Gamma(e)\Gamma(c-e)\Gamma(c-3)\Gamma(a-2)}{2^{2a-2}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a)}\text{x}\ \{(a-1)(a-2)+(e-c)(2c-e-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}+\{(a-1)(a-2)+(e-2)(c-e)\}\frac{\Gamma(c+\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})}\right]}{\int_0^1 x^{c-1}(1-x)^{c-e-1}[1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(a, 3-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Tenth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e-2}[1+\alpha x+\beta(1-x)]^{-2c+e+1} F_1^2\left(a,2-a;\ e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx =$$

$$\frac{\Gamma(e)\Gamma(c-e-1)\Gamma(c-3)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e-1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a-1)}\text{x}\Big[\{-(a-1)(a+1)+c(a+c-2)-e(2c-e-1)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{1}{2}e+\frac{a}{2}-\frac{3}{2})}+\{(a-1)(a-3)-c(c-a)+e(2c-e-1)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}\Big]\text{.........................(4.2.10)}$$

Provided Re(c-3)>0,Re(c-e-1)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{\left[\frac{\Gamma(e)\Gamma(c-e-1)\Gamma(c-3)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e-1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a-1)}\text{x}\ [\{-(a-1)(a+1)+c(a+c-2)-e(2c-e-1)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{1}{2}e+\frac{a}{2}-\frac{3}{2})}+ +\{(a-1)(a-3)-c(c-a)+e(2c-e-1)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}\right]}{\int_0^1 x^{c-1}(1-x)^{c-e-2}[1+\alpha x+\beta(1-x)]^{-2c+e+1}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, 2-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Eleventh Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e-3}[1+\alpha x+\beta(1-x)]^{-2c+e+2} F_1^2(a, 1-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-3)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x[\{-(a+2)(a-3)+3c(c-3)-e(3c-e-4)\}\frac{\Gamma(c-1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{1}{2}e+\frac{a}{2}-1)}+\{-(a+1)(a-2)+c(c-3)+e(e-c)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})}] \quad \text{.........................(4.2.11)}$$

Provided Re(c-3)>0,Re(c-e-2)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{[\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-3)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a))\Gamma(2c-e-a-2)} x [\{-(a+2)(a-3)+3c(c-3)-e(3c-e-4)\}\frac{\Gamma(c-1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{1}{2}e+\frac{a}{2}-1)}+\{-(a+1)(a-2)+c(c-3)+e(e-c)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})}]}{\int_0^1 x^{c-1}(1-x)^{c-e-3}[1+\alpha x+\beta(1-x)]^{-2c+e+2}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, 1-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Twelfth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e-3}[1+\alpha x+\beta(1-x)]^{-2c+e+2} F_1^2(a, -a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-2)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x[\{-(a+1)(a+2)+a(c-e)+c(c-3)\}\frac{\Gamma(c-1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-1)}+\{-(a+1)(a-2)-a(c-e)+c(c-3)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{1}{2})}] \quad \text{.........................(4.2.12)}$$

Provided Re(c)>2,Re(c-e-2)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$F(x)=\frac{[\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-2)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a))\Gamma(2c-e-a-2)} x [\{-(a+1)(a+2)+a(c-e)+c(c-3)\}\frac{\Gamma\left(c-1-\frac{e}{2}-\frac{a}{2}\right)\Gamma\left(\frac{e}{2}-\frac{a}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{e}{2}+\frac{a}{2}+\frac{1}{2}\right)\Gamma\left(c-\frac{e}{2}+\frac{a}{2}-1\right)}+\{-(a+1)(a-2)-a(c-e)+c(c-3)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{1}{2})}]}{\int_0^1 x^{c-1}(1-x)^{c-e-3}[1+\alpha x+\beta(1-x)]^{-2c+e+2}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, -a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Thirteen Formula

$$\int_0^1 x^{c-1}\,(1-x)^{c-e-3}[1+\alpha x+\beta(1-x)]^{-2c+e+2} F_1^2(a,-1;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})dx = \frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-1)}{2^{2a+2}(1+\alpha)^{c}(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} \text{x}[\{-(a-1)(a+2)+(c-1)(2c-e)-2c\}\frac{\Gamma(c-1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2})}+\{-a(a+1)+e(c-1)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+1)\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{1}{2})}] \text{..........................(4.2.13)}$$

Provided Re(c)>1,Re(c-e-2)>0, Re(e)>0. Also the constants α and β are such that none of the expressions 1+α,1+β and 1+αx+β(1-x), 0≤ x ≤1, is not zero.
So by well known definition of probability distributions we have:

$$\text{F(x)=}\frac{[\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-1)}{2^{2a+1}(1+\alpha)^{c}(1+\beta)^{c-e-2}\Gamma(e-a))\Gamma(2c-e-a-2)}\text{x}\ [\{-(a-1)(a+2)+(c-1)(2c-e)-2c\}\frac{\Gamma(c-1-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2})}+ +\{-a(a+1)+e(c-1)\}\frac{\Gamma(c-\frac{1}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}+1)\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{1}{2})}]}{\int_0^1 x^{c-1}\,(1-x)^{c-e-3}[1+\alpha x+\beta(1-x)]^{-2c+e+2}dx}$$

=0, elsewhere

=1, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2(a, -1-a;\ e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

References

[1] M. A. Chaudhry, A. Qadir, M. Rafique and S. M. Zubair, Extension of Euler's beta function, J. Comput. Appl. Math. 78 (1997), no. 1, 19–32.
[2] M. A. Chaudhry, A. Qadir, H. M. Srivastava and R. B. Paris, Extended hypergeometric and confluent hypergeometric functions, Appl. Math. Comput. 159 (2004), no. 2, 589–602.
[3] A. Erdélyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, Higher Transcendental Functions. Vol. I, McGraw-Hill Book, New York, 1953.
[4] A. Erdélyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, Tables of Integral Transforms. Vol. II, McGraw-Hill Book, New York, 1953.
[5] A. Erdélyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, Higher Transcendental Functions. Vol. III, McGraw-Hill Book, New York, 1955
[6] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, 7th ed., Elsevier/Academic, Amsterdam, 2007.
[7] N. Khan, T. Usman and M. Aman, Extended beta, hypergeometric and confluent hypergeometric functions, Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. 39 (2012), no. 1, 83–97

[8] D. Kumar, Solution of fractional kinetic equation by a class of integral transform of pathway type, J. Math. Phys. 54 (2013), no. 4, Article ID 043509.
[9] A. M. Mathai and R. K. Saxena, On a generalized hypergeometric distribution, Metrika 11 (1966), 127–132.
[10] A. M. Mathai, R. K. Saxena and H. J. Haubold, The *H*-Function, Springer, New York, 2009.
[11] T. Pohlen, The Hadamard product and universal power series, Ph. D. thesis, Universität Trier, 2009,
[12] E. D. Rainville, Special Functions, The Macmillan, New York, 1971. [14] M. Shadab, S. Jabee and J. Choi, An extended beta function and its applications, Far East J. Math. Sci. 103 (2011), no. 1, 235–251.