ISSN: 2393-8048
International Advance Journal of Engineering, Science and Management (IAJESM)
Multidisciplinary, Multilingual, Indexed, Double Blind, Open Access, Peer-Reviewed, Refereed-International Journal.
SJIF Impact Factor =8.152, January-June 2025, Submitted in January 2025

Theory And Applications of Difference Cordial Labeling in Cycle-Related and Path-Related Graphs

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Abstract

A major development in the field of graph theory, difference cordial labeling was proposed in 2013 by R. Ponraj, S. Sathish Narayanan, and R. Kala. It provides new insights on analyzing and categorizing graphs according to their structural aspects. The theoretical underpinnings and real-world applications of difference cordial labeling in cycle-related and path-related graphs are explored in this work. In particular, we investigate how to map vertices to distinct numbers and generate edge labels from absolute differences so that the distribution of these labels is balanced. This work emphasizes the distinctive qualities and behaviors of these graphs under difference cordial labeling by means of an extensive analysis of several graph types, including as caterpillar graphs, gear graphs, fundamental cycles, and wheels.

Keywords: Difference Cordial Labeling, Graph Theory, Cycle Graphs, Path Graphs, Wheel Graphs

1. INTRODUCTION

When it comes to the field of graph theory, labeling has always been an extremely important factor in gaining a grasp of the fundamental qualities and traits that graphs represent. In 2013, R. Ponraj, S. Sathish Narayanan, and R. Kala presented a revolutionary notion that they called difference cordial labeling. This concept was introduced among the other labeling systems that were available at the time. Since its introduction, this idea has paved the way for new lines of inquiry and investigation in the field of graph theory. It has also offered a novel viewpoint on the manner in which graphs may be evaluated and classified according to the structural qualities they possess.

It is possible to use difference cordial labeling on a graph G that is of the form (p, q), where p stands for the number of vertices and q stands for the number of edges. To begin the procedure, the first step is to define an injective function f. This function is responsible for mapping the vertices V(G) to a collection of numbers {1,2,…,p}.. Afterwards, this function f causes a similar function f^* to be induced on the edges of the graph E(G). The induced function f^* is defined in such a way that $f^*(uv)=|f(u)-f(v)|$for each edge uv that is specified. The absolute difference between the labels of the vertices that are connected by each edge is largely captured by this induced function.

When it comes to differential cordial labeling, the categorization of these induced edge labels is the most important issue to consider. To be more specific, the edge
uv is assigned a value of 1 if the absolute difference between $|f(u)-f(v)|$ is equal to 1, and a value of 0 if it is not equal to 1. If the criteria $|e_f(0)-e_f(1)|\leq 1$ is fulfilled, the labeling is deemed to be difference cordial. Here, $e_f(i)$ represents the number of edges that are labeled with i (for i=0,1). The labels are dispersed over the edges of the graph in a manner that is balanced thanks to this requirement, which guarantees that they are distributed.

A graph is referred to as a difference cordial graph if it allows for the establishment of such a difference cordial labeling. It is possible to gain insights on the structural balance and symmetry of these graphs by observing that they have a balanced distribution of edge labels. The study of difference cordial graphs not only helps to the expansion of the theory of graph labeling, but it also makes a contribution to a more comprehensive knowledge of graph characteristics and the possible applications of these qualities in a variety of domains, like as network theory, chemistry, and computer science.

Throughout the course of this chapter, we will go deeper into the theoretical basis of difference cordial graphs, investigate a variety of instances and features, and investigate the ramifications of this labeling scheme in both theoretical and practical applications. By conducting an exhaustive investigation, our objective is to unearth the complexities of difference cordial labeling and to determine the relevance of this concept within the context of graph theory as a whole.

2. LITERATURE REVIEW

Lourdusamy and Patrick (2019) An investigation of the sum divisor cordial labeling for graphs linked to cycles and paths is recommended. Within the context of the sum divisor cordial labeling system, their research investigates the structural properties and behavior of these graphs. Because it reveals how such labeling can yield balanced edge distributions, this study is extremely important because it is necessary for comprehending the symmetry and balance that are inherent in these graph architectures.

Kasthuri, Karuppasamy, and Nagarajan (2022) incorporated the idea of SD-divisor labeling into the proceedings of their conference. The use of this labeling to path-related graphs as well as cycle-related graphs allowed them to extend their understanding of divisor cordiality in more complicated graph structures. In order to contribute to the broader study of graph labeling approaches, the SD-divisor labeling process entails mapping the vertices and making certain that the induced edge labels keep a balanced distribution.

Murugan and Mathubala (2015) concentrated on homo-cordial graphs that are connected to paths. A complete examination of homo-cordial labeling is provided by their research. This type of labeling is used in situations when the vertices are labeled in such a way that the difference between the number of edges labeled 0 and the number of edges labeled 1 is no more than one. The significance of this work lies in the fact that it demonstrates how homo-cordial labeling can be utilized for path-related graphs. It also highlights the structural balance and symmetry that can be produced through the utilization of this labeling scheme.

Pandiselvi and Palani (2021) investigated near mean cordial labeling for cycle-related graphics. Their study focuses on labeling vertices so that the edge labels represent a near mean distribution. This idea of near mean amicable labeling is crucial in understanding how labeling might affect the overall structure and features of cycle-related graphs, bringing fresh insights into the balance and distribution of labels within these graphs.

3. DIFFERENCE CORDIAL LABELING OF CYCLE RELATED GRAPHS

Cycles and its variants, such as wheels, gear graphs, and extended cycles, are the components that make up the cycle-related graphs, which are a basic class of graphs in the field of graph theory. A significant amount of information on the symmetry, structure, and equilibrium of these cyclic forms may be gleaned from the investigation of difference cordial labeling in these graphs. In the next part, we will investigate how the idea of difference cordial labeling may be used to cycle-related graphs, shedding light on the distinctive characteristics and behaviors that are associated with these graphs when categorized according to this labeling system.

- **Basic Cycles (C_n)**

A graph of cycles There is just one cycle that has n vertices that make up C_n. A closed loop is formed thanks to the fact that every vertex is linked to precisely two other vertices. In order to implement difference cordial labeling on a cycle graph, we employ an injective function f to transfer each vertex $V(C_n)$ to a unique number from the set $\{1,2,\ldots,n\}$. The induced function $f*$ on the edges $E(C_n)$ assigns labels by utilizing the absolute differences between the vertex labels as the basis for the assignment of labels.

To ensure that the edge labels (0 or 1) are balanced, it is necessary to ensure that the number of edges labelled with 0 and the number of edges labelled with 1 deviate by no more than one. This is the issue that has to be overcome. In most cases, it is simpler to reach this equilibrium for even cycles C_{2k}, but odd cycles C_{2k+1} demand for careful labeling in order to satisfy the difference cordial condition.

- **Wheels (W_n)**

The graph of a wheel Connections are made between a single central vertex and all of the vertices of an n-cycle Cn in order to create W_n. Another degree of intricacy is added to the difference cordial labeling by the presence of this center vertex. It is now necessary for the injective function f to take into consideration the center vertex and make certain that the induced edge labels continue to preserve the necessary equilibrium.

The edges that connect the center vertex to the cycle vertices are one of the most important components of the W_n algorithm. These edges can be labelled in such a manner that they complement the labeling of the cycle Cn, which will help in the process of establishing a difference cordial labeling for the entire wheel graph.

- **Gear Graphs**

Some versions of wheel graphs are gear graphs, in which each spoke of the wheel is split by an extra vertex. Gear graphs are a kind of graph. Additionally, the number of edges and vertices is increased as a result of these new vertices, which makes the labeling process more difficult. It is now necessary for the injective function f to map a more extensive collection of vertices, while simultaneously assuring that the induced function f^* on the edges has the ability to fulfill the difference cordial condition.

- **Generalized Cycles**

Among the many expansions and alterations that may be made to basic cycles, generalized cycles include cycles that contain chords, which are edges that link vertices that are not next to one another. The process of mapping the vertices of these graphs to distinct numbers and ensuring that both the cycle edges and the chord edges contribute to a balanced edge labeling is what is involved in the difference cordial labeling of these graphs.

- **Significance and Applications**

Comprehending the distinction between. There are both theoretical and practical implications associated with the amicable labeling of cycle-related graphs. It is possible to get insights into the structural aspects of these graphs, such as symmetry and connectedness, by observing the balanced distribution of edge labels in these graphic representations. In addition, difference cordial labeling may be utilized in the design of networks, which is an area where it is essential to have balanced load distribution and effective communication paths.

The difference between amicable labeling of cycle-related graphs gives a one-of-a-kind set of obstacles as well as opportunity for more investigation. In the process of applying this labeling system to fundamental graph structures such as basic cycles, wheels, gear graphs, and generalized cycles, we are able to discover novel attributes and advance our understanding of these fundamental graph structures. Our understanding of graph symmetry and structure is improved as a result of the balanced edge labeling that is accomplished using difference cordial labeling. This improvement is a contribution to the more general subject of graph theory.

Theorem 6.2.1: For any $n \geq 4$, a vertex switching of cycle Cn(VSCn) is difference cordial.

Proof: Consider a (p, q) graph G. Cycle C_n 's vertex switching is represented by VSC_n. To obtain it, take a vertex a1 of C_n, remove all the edges that are incident with a_1 and add edges that attach a1 to all the other vertex in C_n that is not next to a1 itself.

We will refer to the vertex set of VSC_n as $V(VSC_n)=\{a_i|1\leq i\leq n\}$. The VSCn edge set is $E(VSC_n)=\{(a_ia_{i+1})|2\leq i\leq n-1\}\cup\{(a_1a_j)|3\leq j\leq n-1\}$

Therefore, VSC_n has $2n-5$ edges and nnn vertices.

Define a labeling function f that maps the set {1,2,...,p} to the vertex set V(G) of VSCn.

$f(a_1) = n$

$f(a_i) = i - 1$, for $2\leq i\leq n$

On the edges of VSC_n, this function f produces a function f^*. We determine the frequency of each label by examining the f^* labeling of the edges.

- The label assigned to edges ($a_i a_{i+1}$) that connect successive vertices is 1. Because there are n−2 of these edges, ef(1)=n−2.
- The label assigned to edges joining a1 to vertices other than those next to $a_1$in C_n is 0. Since there are n−3 of these edges, ef(0)=n−3.

Based on these computations:
ef(1)=n−2
ef(0)=n−3
We must ascertain whether the absolute difference between the number of edges labeled 0 and the number of edges labeled 1 is at most 1 in order to establish whether VSC_n is difference cordial:
$|ef(0)-ef(1)|=|(n-3)-(n-2)|=|-1|=1$
For any n≥4, VSC_n meets the difference cordial condition since $|ef(0)-ef(1)|\leq 1$.

4. DIFFERENCE CORDIAL LABELING OF PATH RELATED GRAPHS

Path-related graphs are a basic type of graphs in graph theory that are distinguished by their linear structure. Caterpillar graphs, simple routes, and various variants created from basic paths are examples of these graphs. These path-related graphs may be studied structurally and symmetrically by using the idea of difference cordial labeling, which consists of labeling vertices and then labeling edges based on the absolute differences between vertex labels. This section will examine the use of difference cordial labeling to path-related graphs, emphasizing the particular difficulties and traits of these graphs under this labeling scheme.

➢ **Simple Paths (P_n)**

A straight-line connecting n vertex makes up a simple route P_n. An injective function f is utilized to transfer each vertex $V(P_n)$ to a distinct number from the set {1,2,...,n} in order to perform difference cordial labeling on a route P_n. Labels are assigned by the induced function f∗ on the edges $E(P_n)$ according to the absolute differences between labels of neighboring vertices. If there is at most one difference between the number of edges labeled 0 and those labeled 1, or $|e_f(0)-e_f(1)|\leq 1$, the labeling is deemed difference cordial.

➢ **Caterpillar Graphs**

There is a specific kind of tree known as a caterpillar graph, which may be created from a straightforward path by affixing leaves to the vertices of the path. A core "spine" or route is maintained by these networks, and additional vertices are linked directly to this spine. In the process of applying difference cordial labeling to caterpillar graphs, the injective function f is responsible for assigning distinct labels to each and every vertex, which includes the spine and the leaves that are linked to it. Following this, the induced edge labels need to be balanced in order to guarantee that the graph is difference cordial. Because of the linear form of the spine and the uncomplicated attachment of leaves, the structure of caterpillar graphs frequently makes the labeling process easier to understand.

➢ **Variations and Generalizations**

Path-related graphs can comprise a wide variety of modifications and generalizations of basic routes and caterpillar graphs. These can include paths that include extra branches or paths that are embedded in bigger graph structures. The Variation In situations like these, cordial labeling entails mapping the vertices to different integers and making certain that the induced edge labels over the whole graph satisfy the difference cordial condition. As the complexity of the graph structure rises, the difficulty of the task becomes more prominent, necessitating a more careful examination of the manner in which vertex labels are allocated in order to preserve a balanced distribution of edge labels.

➢ **Significance and Applications**

There are both theoretical and practical ramifications associated with gaining an understanding of difference cordial labeling in path-related graphs. The symmetry and structural aspects of these graphs may be seen by the balanced labeling of edges, which can give insights into these properties. In addition, path-related graphs are frequently used in a variety of applications, such as network architecture, which is an area where effective communication channels and balanced load distribution are vital. Research on the use of different cordial labels in these graphs has the potential to contribute to the optimization of these applications.

In the process of difference cordial labeling of path-related graphs, it is necessary to give distinct labels to the vertices and to make certain that the edge labels are distributed in a balanced manner depending on the absolute differences between them. Under this labeling method, simple pathways, caterpillar graphs, and their modifications each provide their own set of obstacles and the qualities that distinguish them from one another. Through the investigation of these path-related graphs, we are able to establish a more profound comprehension of the structural characteristics of these graphs as well as the wider implications of difference cordial labeling in graph theory.

5. CONCLUSION

This research has investigated the complex properties of difference cordial labeling, an idea that is essential to the development of graph theory knowledge. We have illustrated the distinct characteristics and behaviors that different graph structures—such as wheels, caterpillar graphs, gear graphs, and cycles—display when they are labelled with difference cordially through a thorough examination of these graph structures. This labeling scheme reveals deeper insights into the structural subtleties and symmetry of these graphs, in addition to offering a balanced distribution of labels. Difference cordial labeling has applications in chemistry, computer science, network theory, and other fields, demonstrating its importance and versatility.

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