

## Mathematical Model to Predict Ignition Delay & Heat Transfer During Combustion from a Direct Injection Diesel Engine: Part-II

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### Abstract.

Tighter emission norms, fewer accurate ignition delay prediction & heat transfer models forces to develop a numerical model which actively control both the phenomena to increase the efficiency of diesel engine. This research work is focused on the simulation and experimental investigation of simplified model for prediction of ignition delay along with heat transfer from the cylinder during combustion. The prediction of ignition delay is very crucial for heat release prediction which is used for pressure prediction during combustion. All the models were tested under two operating speed of 2200 rpm and 1400 rpm with varying load. The Watson model gives average deviation of 24%. The increase in ignition delay from 100% to 50% load at 2200 rpm is 33.3% while at 1400 rpm it is 14.29%. Ignition delay observed to be higher at part load because mass of fuel consumed during premixed phase burning increased. Due to decrease in load on engine at particular rpm the rate of heat release decreases but when load is constant and engine speed decreases then rate of heat releases increases.

**Keywords:** Direct injection diesel engine, mathematical model, ignition delay, Watson model, Hohenberg's correlation.

### Introduction

Diesel engines have been widely used for heavy duty vehicles such as trucks, buses, tractors, cranes, cars, etc., due to their higher power to weight ratio [1]. To reduce the research time and cost simulation models need to be investigated into full-cycle diesel engines applications. Large number of simulation models were developed by the various researchers included two-zone models [2] but zero-dimensional single zone combustion model is the simplest one [3, 4], in which diesel-air mixture is assumed to be uniform in composition and temperature at any instant during combustion phase. Zero dimensional single zone models are used for ignition delay and heat release with high accuracy [5] which required pressure-crank angle data as input data [3-6]. Ericson et al. [7] predicted ignition delay using a linear black-box model and gross heat release rate (GHRR). The accurate heat release model developed by Watson et al. [8] which can take care of both premixed and diffusive combustion. Annand correlation was used for heat loss from the cylinder during combustion [9, 10]. The main limitation of single-zone model is that it is inferior to estimate the spatial temperature distribution and hence cannot predict adiabatic flame temperature and emissions [6, 11]. 3D models require huge computational power, cost, and time compared to 1D [12]. Hence, 0D with multi-zone models are proposed for simulation of species concentration along with other parameters.

Chmela et al, [27] worked on non-dimensional combustion model that relies on the concept of mixing controlled combustion (MCC Heat Release Rate) avoiding the detailed description of the individual mixture formation and fuel oxidation processes. It can be shown that the rate of heat release (ROHR) was controlled mainly by the two items, i.e. the instantaneous fuel mass present in the cylinder charge and the local density of turbulent kinetic energy. In this study neither the geometry of the combustion chamber nor the processes of spray development, evaporation and mixture formation were taken into account. Due to the simple physical law used, the simulation is limited to the diffusion part of the total heat release rate history.

The main objective of this study is to develop mathematical modelling to simulate the ignition delay period by considering both premixed and diffusion combustion along with heat transfer during combustion.

### Predictive Heat Release model



The heat release model is of the predictive type, i.e. in cylinder pressure measurements are not needed as inputs. Figure 1 show the procedure for NOx prediction model.

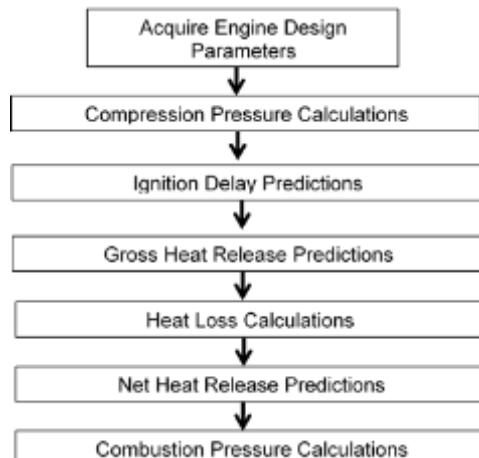


Figure 1. Procedure for ignition delay and heat transfer prediction model.

### Experimental Setup and Methodology

Figure 2 shows experimental setup of turbocharged diesel engine which is coupled with eddy current dynamometer. The engine specification is provided in Table 1 which is identical setup which was discussed in part-I [14].

Table 1: Engine specification

Parameter	Specifications
Engine Model	4- cylinder Turbocharged DI Diesel
Bore (mm)/ Stroke (mm)	102/ 110
Connecting rod length (mm)	220
IVC (CA)/ EVO (CA)	210/ 540
Rated speed (rpm)	2200
Compression ratio/ Swept volume (liter)	17.4/ 0.9 per cylinder
Injection system	Common rail

The engine suction side is fitted with air-conditioning system, air measuring system, and fuel measuring unit while exhaust side is partially connected to emission analyzer. AVL Indiset, was used as data acquisition system.

Un-cooled miniature pressure transducer (GM12D); AVL 365C angle encoder; “FEV Fuelrate (FI-15)”, ABB sensy flow meter were used for pressure measurement, crank angle position, fuel flow rate and air induction rate. Yantrashilpa’s coolant condition unit YS4025 installed to maintain the engine water temperature within band of  $\pm 0.2$  deg. C under specific test conditions.

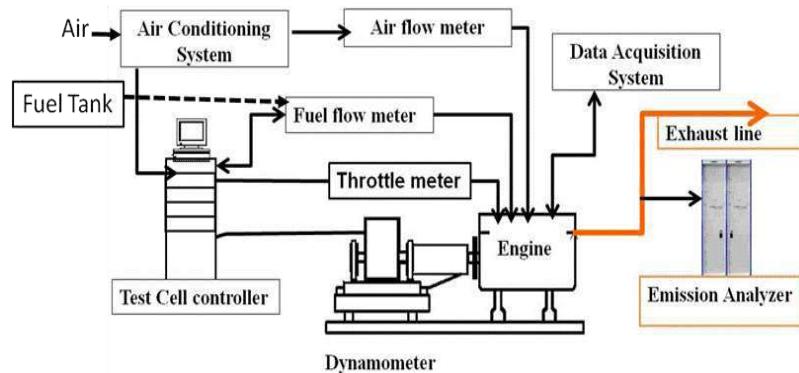


Figure 2. Engine experimental setup to collect data for validation of simulated results [14]  
**Methodology for Ignition Delay**

The combustion process in a compression ignition engine is a premixed-diffusion one. Arrhenius type equations are generally used for ignition delay calculation, as indicated by equation:

$$\tau_{id} = a p^{-n} \exp\left(\frac{E_a}{R T}\right) \quad (1)$$

Where, a, n are constants.

The value of constants chosen in Arrhenius type equation by Wolfer and Watson are given in table 2:

Table 2: Value of constants for Arrhenius equation

Constants	By Wolfer [1]	By Watson [4,6]
a	0.44	3.45
n	1.19	1.02
E <sub>a</sub> /R	4650 K	2100 K

Heywood recommends a correlation for ignition delay developed by Hardenberg and Hase (1979)[15];

$$\tau_{id} = (0.36 + 0.22 v_p) X1. Y1 \quad (2)$$

Where,

$$X1 = \exp\left(E_a \left(\frac{1}{R_0 T} - \frac{1}{17190}\right)\right),$$

$$Y1 = \{21.2/(p - 12.4)\}^{0.6}$$

$$E_a = \{1310000/(CN + 25)\}$$

In view of the fact that the cylinder pressure and temperature are considerably varied during the ignition delay period, the following equation has been developed to account for these changing conditions [16]:

$$\int_{tsoi}^{tsoc} \left(\frac{dt}{\tau_{id}}\right) = 1 \quad (3)$$

The ignition delay time can be evaluated once the above relationship is satisfied:

$$t_{delay} = t_{soc} - t_{soi} \quad (4)$$

### Combustion Model

Watson model [16] is used for combustion modelling. Total combustion process is mainly divided into two major parts; first premixed combustion and second is diffusion combustion. This premixed phase is a consequence of the mixture prepared during the ignition delay period burning rapidly; the diffusion burning phase accounts for the remainder of combustion.

$$m_{fb}(t) = \beta f_{pre}(t) + (1 - \beta) f_{diff}(t) \quad (5)$$

Where,

$$\beta = 1 - \{(a \emptyset)/\tau^c\},$$

$$f_{pre(t)} = \{1 - (1 - t^{k_1})\}^{k_2},$$

$$f_{diff(t)} = 1 - \exp(-k_3 \times (t)^{k_4})$$

In equation (5), t is the time measured from ignition, non-dimensionalised by the total combustion time:

$$t = \{t_\theta - t_{ign}\}/\{\Delta t_{comb}\} \quad (6)$$

The best-fit values of these parameters for a range of DI diesel engines, based on nominal combustion duration of 125°CA, were found to be

$$k_1 = 2.0 + 1.25 \times 10^{-8} (\tau_{id} N)^{2.4} ;$$

$$k_2 = 5000;$$

$$k_3 = 14.2/\{\emptyset\}^{0.664} ;$$

$$k_4 = 0.79 \times \{k_3\}^{0.25} ;$$

$$0.8 < a < 0.95;$$

$$0.25 < b < 0.45;$$

$0.25 < c < 0.50$ .

#### 4.3.5 Heat Transfer calculation

The instantaneous heat transfer across the walls for engines was estimated using Hohenberg's correlation (1979)[16];

$$h_c = \frac{130 p_c^{0.8} (v_p + 1.4)^{0.8}}{V^{0.06} T_g^{0.4}} \quad (7)$$

Convective heat transfer [16]

$$\frac{dQ_{ht}}{d\theta} = h_c A_c (T_{cyl} - T_w) \left( \frac{1}{6n} \right) \quad (8)$$

Cylinder pressure or combustion pressure is calculated using the following correlation:

$$\frac{dp}{d\theta} = h_c \cdot A_s \left( \frac{dQ_n}{d\theta} - \frac{\gamma}{\gamma-1} p \frac{dV}{d\theta} \right) \left( \frac{\gamma-1}{V} \right) \quad (9)$$

#### Net Heat Release

The equation for the apparent heat release rate can be determined from the pressure data, is given by;

$$\frac{dQ_n}{dt} = \frac{dQ}{dt} - \frac{dQ_L}{dt} = \frac{\gamma}{\gamma-1} p \frac{dV}{dt} + \frac{1}{\gamma-1} V \frac{dp}{dt} \quad (10)$$

The ratio of specific heats ( $\gamma$ ) is not necessarily constant through the compression, combustion, or expansion process. The ratio is calculated by a relatively simple correlation [16] as follows:

$$\gamma = 1.338 - 6 \times 10^{-5} T + 1 \times 10^{-8} T^2 \quad (11)$$

#### Results and discussion

**Effect of load on Ignition delay:** Figure 3 and 4 shows the prediction result obtained for ignition delay in Crank angle using different expressions given by Wolfer, Watson, and Hardenberg. The Hardenberg expression is quite relevant and most accurate or we can say that ignition delay prediction using this model is very close to experimental results. In first step, ignition delay is predicted at an engine speed of 2200 rpm with varying loads as shown in figure 3.

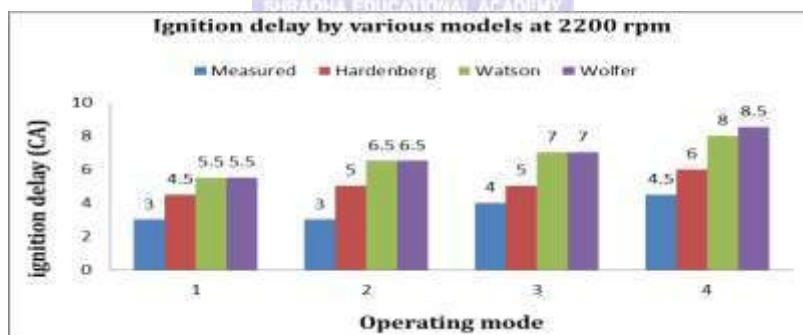


Figure 3: Ignition Delay period using different models at 2200 rpm

In second step, ignition delay is predicted and compared with measured values. The results of second trial is given in figure 4.



Figure 4: Ignition Delay period using different models at 1400 rpm

Measured ignition delay using Watson model shows good agreement with predicted values. As load decreases, exhaust gas temperature decreases with the leaner engine operation. With less energy to be recovered in comparison to high load in the turbocharger, the resulting boost pressure decreases. Accordingly, cylinder pressure and temperature during the ignition delay period decreases. At 2200 rpm and 100% operating load, the peak pressure is 120 bar, ignition delay is 5.5 degree CA while at seventh mode (10% load) peak pressure and ignition delay are 65 bar and 8 degree CA respectively. Similarly, at 1400 rpm and 100% load, the peak pressure is 103.4 bar and ignition delay is 5 degree CA while at 10% load these values are 76.6 bar and 6 degree CA. From this we conclude that at low pressure and temperature with low load eventually increases the ignition delay period.

**Effect of load on Heat Release:** The results obtained at various operating modes during simulation of heat release rate are discussed in the following paragraphs.

The Watson model is used for heat release calculation. To verify the accuracy of different operating mode, experimental data of heat release is compared with predicted model. The observed cylinder pressure profiles reflect the effects of in-cylinder heat release, heat transfer to the cylinder surfaces and work transfers. Even though predictions of the gross heat release rate are directly available from the model, the apparent heat release rates have been calculated so as to compare them with experimental data. The direct comparison of heat release rate plot from experimental with numerical results is not possible. In figures 5 to 11 the predicted apparent heat release rates are plotted at different operating conditions.

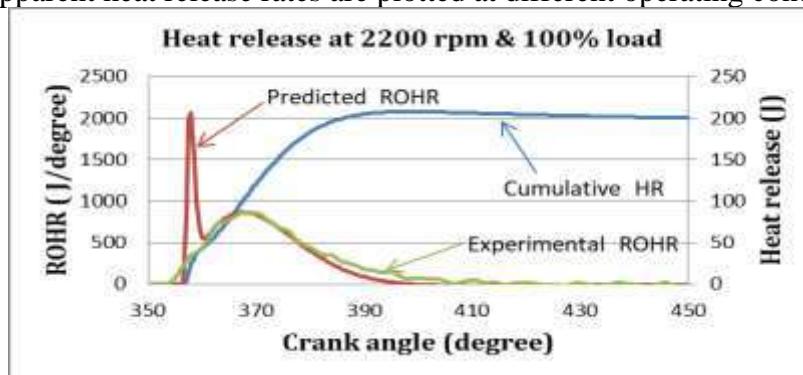


Figure 5: ROHR predicted and cumulative heat release at 2200 rpm and 100% load  
 Figure 5 shows the results of first operating mode (at 100% load and 2200 rpm), first peak of the heat release rate is predicted to occur at 358 degrees CA while the second one occurs at 367.5 degrees CA. The magnitude of maximum ROHR in premixed combustion is 206 J/degree while during diffusion combustion is 86.6 J/degree. Correspondingly, experimental data indicate that the diffusion peak occurs at 368 degrees CA and ROHR was 86 J/degree. The mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 50.91 mg per cycle, 0.5099 and 28.83 respectively. Due to higher inlet pressure and temperature at inlet condition (2 bar and 378.3 K) the pressure and temperature at SOC are 80 bar and 925.6 K which increases the fraction of fuel burned in premixed combustion phase is 20.938% by mass. Rest of fuel mass is burned in diffusion phase which starts at 361 degree CA.

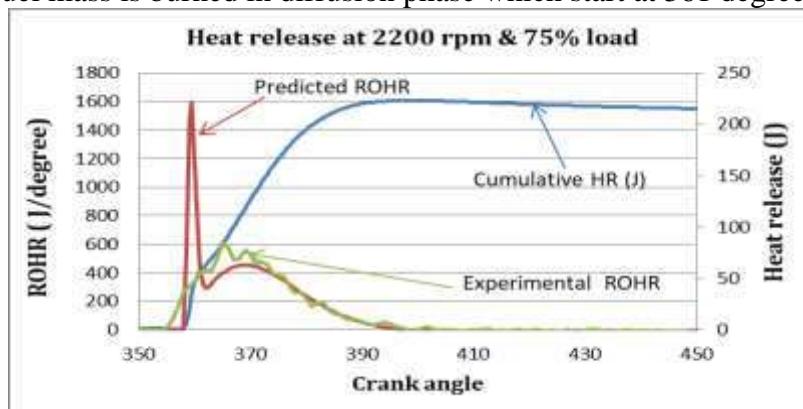


Figure 6: Rate of heat Release predicted and cumulative heat release at 2200 rpm and 75% load

In second operating mode, load on engine is lowered from 100% to 75% at the same speed of 2200 rpm than the two peak values of ROHR are 221.21 J/degree and 63.1 J/degree and the corresponding CA in degree are 359.5 degree CA and 369 degree CA. Both the predicted and experimental results are shown in figure 6. The measured peak value of ROHR during diffusion combustion was 84 J/degree at 365 degree CA. In this operating mode mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 39.24 mg per cycle, 0.4366 and 33.67 respectively.

In second operating mode, pressure and temperature both at inlet and SOC are 1.65 bar, 358.9 K and 62 bar, 876.8 K respectively. Both temperatures and pressures are lower compared to first mode results increase the ignition delay period from 5.5 degree CA to 6.5 degree CA and increase the maximum ROHR from 206 J/degree to 221.21 J/degree. This happen due to higher ignition delay and results more percentage of fuel mass exactly 26.754% (or 12.0944 mg consumed out of 39.24 mg injected quantity during this cycle) is consumed during the premixed combustion. Diffusion combustion starts at 362.5 degree CA instead of 361 degree CA in first mode.

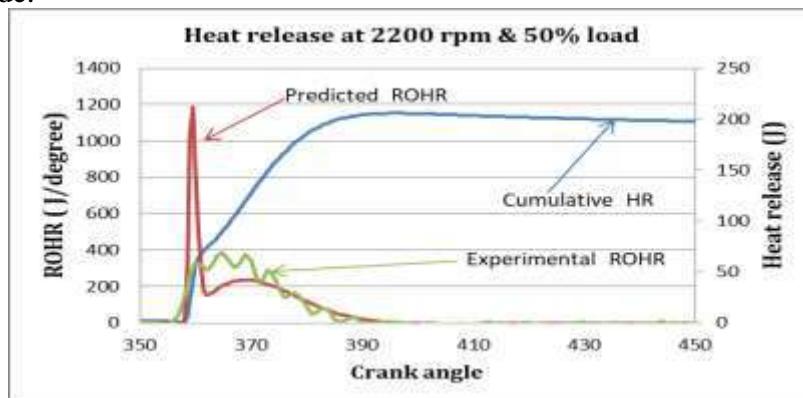


Figure 7: Rate of heat Release predicted and cummulative heat release at 2200 rpm and 50% load

In third operating mode during experiment, the load on engine was 50% at a speed of 2200 rpm. The mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 28.11 mg per cycle, 0.3521 and 41.75 respectively. The results obtained is shown in figure 7. The magnitude of maximum ROHR in premixed combustion is 211.63 J/degree at 359 degree CA while during diffusion combustion is 42.18 J/degree at 369 degree CA respectively. Correspondingly, experimental data indicate that the diffusion peak occurs at 364 degrees CA and ROHR was 67 J/degree.

In this operating mode, pressure and temperature both at inlet and SOC are 1.48 bar, 358.9 K and 60.7 bar, 888.6 K respectively. Both temperatures and pressures are lower compared to first mode results increase the ignition delay period from 5.5 degree CA to 7 degree CA and increase the maximum ROHR from 206 J/degree to 211.63 J/degree. This happen due to higher ignition delay and results more percentage of fuel mass exactly 37.57% ( or 10.5061 mg consumed out of 28.11 mg injected quantity during this cycle) is consumed during the premixed combustion. Diffusion combustion start at 362.5 degree CA instead of 361 degree CA in first mode.

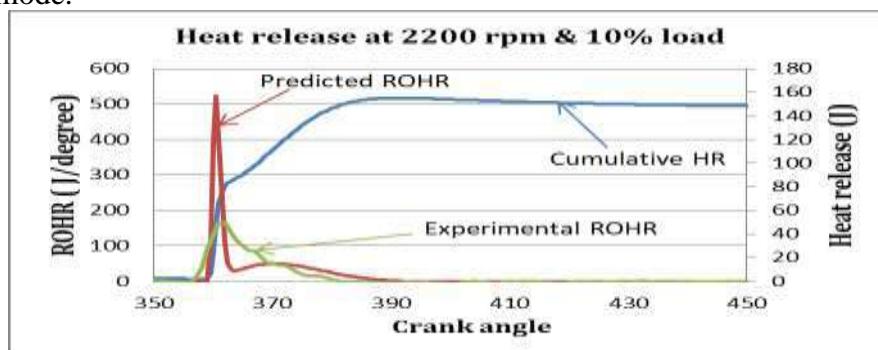


Figure 8: Rate of heat Release predicted and cummulative heat release at 2200 rpm and 10% load

In fourth operating mode, the load on engine was kept 10% of rated load at 2200 rpm than the two peak values of ROHR are 156.95 J/degree and 14.82 J/degree and the corresponding CA in degree are 360.5 degree CA and 370.5 degree CA. Both the predicted and experimental results are shown in figure 8. The measured peak value of ROHR during diffusion combustion was 49 J/degree at 361 degree CA. The mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 12.61 mg per cycle, 0.1818 and 80.87 respectively.

In this operating mode, pressure and temperature both at inlet and SOC are 1.25 bar, 358.9 K and 51.6 bar, 888.7 K respectively. Both temperatures and pressures are lower compared to first mode results increase the ignition delay period from 5.5 degree CA to 8 degree CA and decrease the maximum ROHR from 206 J/degree to 156.95 J/degree. This happen due to higher ignition delay and results more percentage of fuel mass exactly 52.211% (or 6.58385 mg consumed out of 12.61 mg injected quantity during this cycle which is lower than 10.6595 mg of fuel consumed in premixed combustion) is consumed during the premixed combustion and diffusion combustion start at 364 degree CA instead of 361 degree CA in first mode.

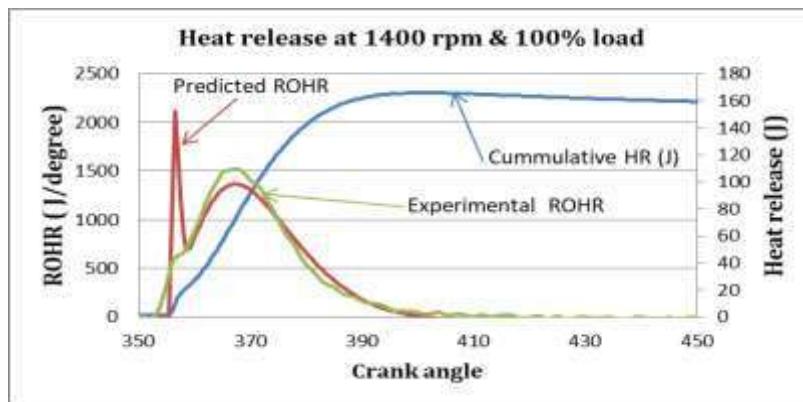


Figure 9: Rate of heat Release predicted and cummulative heat release at 1400 rpm and 100% load

In fifth operating mode, the experiment was conducted at full load but at time engine speed was lowered from 2200 rpm to 1400 rpm. The mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 56.85 mg per cycle, 0.727 and 20.21 respectively. The two peak values of ROHR are 152.77 J/degree and 98.43 J/degree and the corresponding CA in degree are 356.5 degree CA and 367.5 degree CA. Both the predicted and experimental results are shown in figure 9. The measured peak value of ROHR during diffusion combustion was 109 J/degree at 367 degree CA.

In this operating mode, pressure and temperature both at inlet and SOC are 1.43 bar, 337.7 K and 57.5 bar, 837.7 K respectively. Both temperatures and pressures are lower compared to first mode but due to 1400 rpm results decrease the ignition delay period from 5.5 degree CA to 5 degree CA and decreases the maximum ROHR from 206 J/degree to 152.77 J/degree. This happens due to lower ignition delay. The percentage of fuel mass exactly 25.956% (or 14.7562 mg consumed out of 56.85 mg injected quantity) is consumed during the premixed combustion .Diffusion combustion start at 364 degree CA instead of 361 degree CA in first mode.

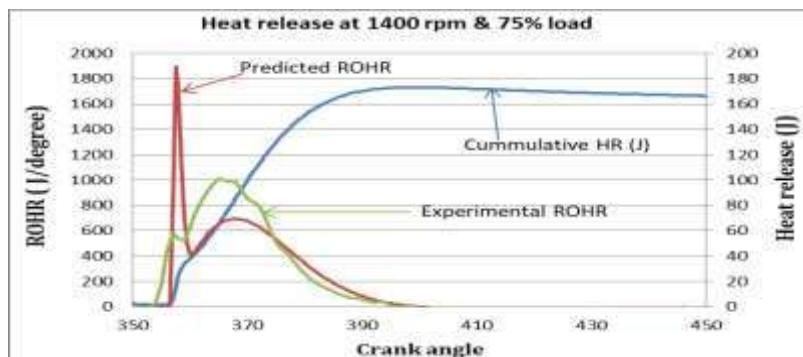


Figure 10: Rate of heat Release predicted and cummulative heat release at 1400 rpm and 75% load

In sixth operating mode, load on engine is lowered from 100% to 75% at the same speed of 1400 rpm than the two peak values of ROHR are 189.7 J/degree and 60.6 J/degree and the corresponding CA in degree are 357.5 degree CA and 368 degree CA. Both the predicted and experimental results are shown in figure 10. The measured peak value of ROHR during diffusion combustion was 101 J/degree at 365 degree CA. In this operating mode mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 42.44 mg per cycle, 0.5843 and 25.16 respectively.

In this operating mode, pressure and temperature both at inlet and SOC are 1.32 bar, 326.3 K and 54.3 bar, 812.2 K respectively. Both temperatures and pressures are lower compared to first mode equal ignition delay period of 5.5 degree CA. The ROHR decreases from 206 J/degree to 189.7 J/degree. This happen due to lower pressure and temperature at SOC which increase chemical delay and results more percentage of fuel mass exactly 29.829% ( or 12.6594 mg out of 42.44 mg) is vaporised during the premixed combustion but not consumed completely and diffusion combustion start at 364 degree CA instead of 361 degree CA in first mode.

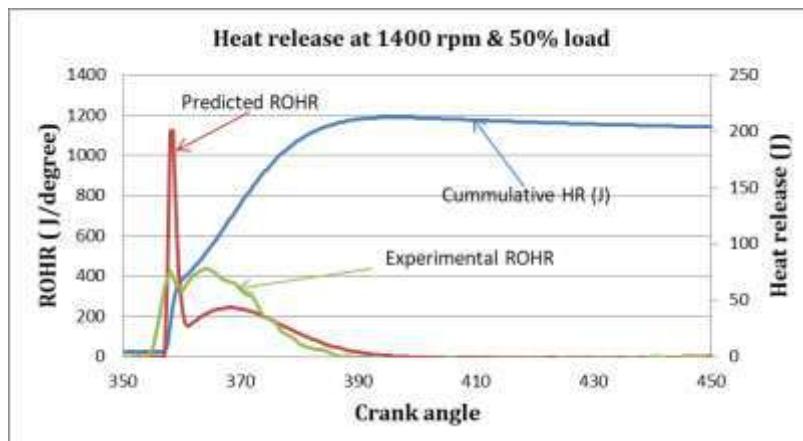


Figure 11: Rate of heat Release predicted and cummulative heat release at 1400 rpm and 50% load

In last operating mode during experiment, the load on engine was 50% at a speed of 1400 rpm. The results obtained is shown in figure 11. The magnitude of maximum ROHR in premixed combustion is 201 J/degree at 358.5 degree CA while during diffusion combustion is 43 J/degree at 368 degree CA respectively. Correspondingly, experimental data indicate that the diffusion peak occurs at 364 degrees CA and ROHR was 78 J/degree. In this operating mode mass of fuel injected, equivalence ratio, and actual A/F ratio measured were 28.87 mg per cycle, 0.4211 and 34.91 respectively.

In this operating mode, pressure and temperature both at inlet and SOC are 1.23 bar, 317.2 K and 51.2 bar, 801.4 K respectively. Both temperatures and pressures are lower compared to first mode results increase the ignition delay period from 5.5 degree CA to 6 degree CA and decrease the maximum ROHR from 206 J/degree to 201 J/degree. This happen due to higher ignition delay and lower pressure and temperature at SOC results more percentage of fuel mass exactly 38.256% ( or 11.0444 mg consumed out of 28.87 mg injected quantity during this cycle) is consumed during the premixed combustion and diffusion combustion start at 364 degree CA instead of 361 degree CA in first mode.

From above results, we conclude that as load on engine decreases the relative importance of the premixed combustion phase increases. This is attributed to the long ignition delay which is primarily caused by (i) low injection rates and low injection line pressures used, thus prolonging the phisical preparation of the fuel-air mixture, (ii) more time per cycle available for heat loss, thus prolonging the chemical aspects of the ignition delay. Since pobably maximum fuel mass is injected during the ignition delay period, this decrease mixing controlled combustion.

### Effect of load on Heat transfer

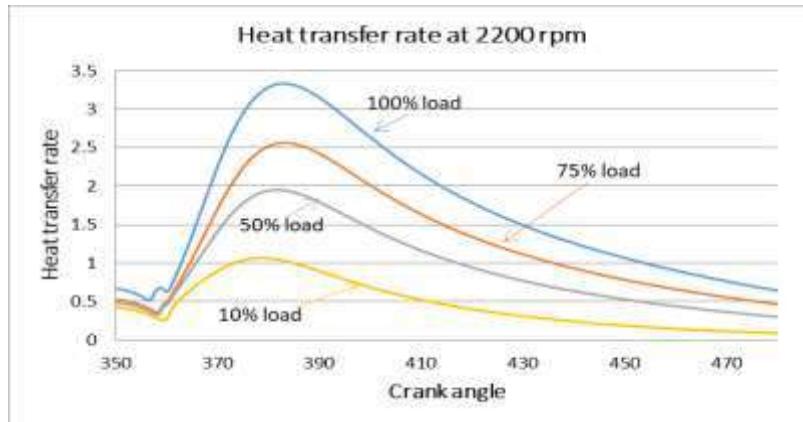


Figure 12: Comparison of heat transfer rate at different load conditions at 2200 rpm

Figure 12 shows prediction of convective heat transfer rate over the duration of the expansion process. It is observed that initially before SOC at loads heat transfer is approximately equal but it after TDC the difference between heat transfers quite higher at high load than other load conditions. Table 3 shows the predicted maximum heat transfer rate along with corresponding crank angle. This is because at higher load, pressure and temperature before SOC are higher in comparison to lower load conditions and more heat release generation due to more fuel injection at high load. At 100% load, injected mass of fuel is 50.91 mg while at 50% load it is 10.5061 mg and at 10% load it is 6.58385 mg.

Table 3: Maximum heat transfer rate at 2200 rpm

2200 rpm		
Load	Heat transfer rate (J/CA)	Degree (CA)
100%	3.33	383
75%	2.562	383
50%	1.95	382
10%	1.063	378.5

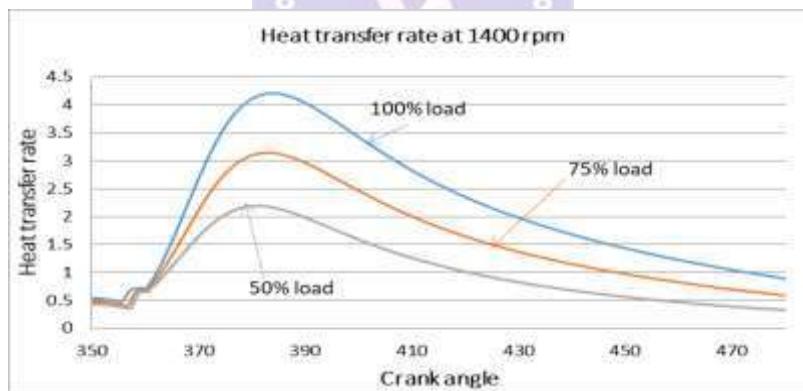


Figure 13: Comparison of heat transfer rate at different load conditions at 1400 rpm at 100% load

Similarly, at 1400 rpm, the heat transfer rate is also predicted at full load, 75% load and 50% load (refer Figure 13) and observations are listed in table 4. The above result also supported the statement that at higher load heat transfer rate is higher. At 100% load, injected mass of fuel is 56.85 mg while at 50% load it is 28.87 mg.

Table 4: Maximum heat transfer rate at 1400 rpm

1400 rpm		
Load on engine	Heat transfer rate (J/degree)	Crank angle (CA)
100%	4.207	383.5
75%	3.14	383
50%	2.19	381

## Conclusions

The ignition delay and heat transfer predicting model is developed with précis accuracy for 4-stroke turbocharged DI diesel engine. In this study, different models were used for in-cylinder predictions. The main findings of the work are summarised below:

- Due to decrease in load on engine at particular rpm the rate of heat release decreases but when load is constant and engine speed decreases then rate of heat releases increases.
- The prediction of ignition delay is very crucial for heat release prediction which is used for pressure prediction during combustion. The Watson model used gives average deviation of 24%. The increase in ignition delay from 100% to 50% load at 2200 rpm is 33.3% while at 1400 rpm it is 14.29%. There is no difference in ignition delay between 100% load and 75% load at both 2200 rpm and 1400 rpm. At part load, low pressure and temperature at SOC results higher ignition delay .
- Due to higher ignition delay at part load the mass of fuel consumed during premixed phase is increases. At 2200 rpm speed, 20.938%, 26.754%, 37.57% and 52.211% of fuel consumed during cycle at 100%, 75%, 50% and 10% load. Similarly at 1400 rpm fuel consumption in premixed phase is 25.956%, 29.829% and 38.256% at 100%, 75%, and 50% load. Therefore, we conclude that due to large fluctuation in load ignition delay increases.

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