Applications Of Frdtm to Solve Fractional Mathematical Models

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ABSTRACT

The Fractional Reduced Differential Transform Method (FRDTM) is a powerful and efficient tool for solving fractional differential equations, which frequently arise in various fields such as biology, physics, and engineering. This paper explores the application of FRDTM to three distinct fractional mathematical models: the transmission of nerve impulses through the Fitzhugh-Naguma equation, the time fractional Rosenau-Hyman equation, and a bio-mathematical model for the evolution of smoking habits in fixed populations. The method's accuracy and effectiveness are evaluated through error analysis and graphical approaches, demonstrating its superiority over traditional methods like the Homotopy Perturbation Method (HPM) and Adomian Decomposition Method (ADM). Numerical results confirm that FRDTM not only provides precise solutions but also significantly reduces computational complexity. This study highlights the potential of FRDTM as a reliable approach for solving complex fractional mathematical models, contributing to advancements in theoretical and applied mathematics.

Keywords: Fractional Reduced Differential Transform Method (FRDTM), Fractional Differential Equations, Fitzhugh-Naguma Equation, Rosenau-Hyman Equation, Bio-Mathematical Model, Error Analysis, Numerical Solutions.

1. INTRODUCTION

Here, we'll look at how the FRDTM may be used to solve biological systems models like the transmission of nerve, the time fractional Rosenau-Hymanequation, and an out-of-the-box model for the change of smoking habits in fixed populations. Error analysis and graphical approaches are used to evaluate the suggested method's accuracy.

2. FRDTM FOR FRACTIONAL MATHEMATICAL MODEL FOR TRANSMISSION OF NERVE

This part uses the FRDTM to solve the Fitzhugh-Naguma fractional equation. When compared to HPM and ADM, the numerical solution found using this approach produces very accurate results. Using the suggested method, the nonlinear fractional partial derivative equations may be solved effectively. We consider the fractional Fitzhugh-Naguma equation as pursue:

$$u^t_\alpha=u_{xx}+u(u-\delta)(1-u),$$

$$u^t_\alpha=u_{xx}+u^2-u^3-u\delta+u^2\delta, \qquad (1.1)$$

where δ is arbitrary constant and $0<\delta\leq 1$.

Research into the transmission of nerve impulses is shown in the equation (1.1). In biology, population genetics, and circuit theory, Equation (1.1) has several applications. The Fitzhugh-Naguma classical equation was solved using HPM, VIM, and ADM.

Solution by FRDTM

We have following recurrence formula for equation (1.1)

$$U_{k+1}(x)=\frac{\Gamma(1+\alpha k)}{\Gamma(\alpha+1+\alpha k)}\left[\frac{\partial^2}{\partial x^2}U_k(x)+(1+\delta)\sum_{r=0}^{k}U_r(x)U_{k-1}(x)-\sum_{r=0}^{k}\sum_{i=0}^{r}U_i(x)U_{r-i}(x)U_{k-r}(x)-\delta U_k(x)\right]$$

with initial conditions (1.2)

$$U_0(x)=\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}x}{4}\right) \qquad (1.3)$$

Using recurrence relation (1.2) and initial condition (1.3), we get for k=0

$$U_1(x)=\frac{1}{\Gamma(\alpha+1)}\left[\frac{\partial^2}{\partial x^2}U_0(x)+(\delta+1)U_0^2(x)-U_0^3(x)-\delta U_0(x)\right]$$

$$U_1(x)=\frac{1}{\Gamma(\alpha+1)}\left[\frac{\partial^2}{\partial x^2}\left(\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}x}{4}\right)\right)+(1+\delta)\left(\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}x}{4}\right)\right)^2\right.$$
$$\left.-\left(\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}x}{4}\right)\right)^3-\delta\left(\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}x}{4}\right)\right)\right]$$

$$U_1(x) = \frac{1}{\Gamma(\alpha+1)}\left[-\frac{1}{8}\sec h^2\left(\frac{\sqrt{2}x}{4}\right)\tan h\left(\frac{\sqrt{2}x}{4}\right) + \left(\frac{1}{2}+\frac{1}{2}\tan h\left(\frac{\sqrt{2}x}{4}\right)\right)\left\{(1+\delta)\left(\frac{1}{2}+\frac{1}{2}\tan h\left(\frac{\sqrt{2}x}{4}\right)\right) - \left(\frac{1}{2}+\frac{1}{2}\tan h\left(\frac{\sqrt{2}x}{4}\right)\right)^2 - \delta\right\}\right]$$

$U_1(x) = \frac{1}{\Gamma(\alpha+1)}\frac{(1-2\delta)}{8}\sec h^2\left(\frac{\sqrt{2}x}{4}\right)$ Similarly we get ,

$$U_2(x) = -\frac{1}{\Gamma(2\alpha+1)}\frac{(1-2\delta)^2}{16}\tanh h\left(\frac{\sqrt{2}x}{4}\right)\sec h^2\left(\frac{\sqrt{2}x}{4}\right)$$

Therefore the approximate solution of (1.1) is known as

$$U(x,t) = \left(\frac{1}{2}+\frac{1}{2}\tan h\left(\frac{\sqrt{2}x}{4}\right) + \frac{1}{\Gamma(\alpha+1)}\frac{(1-2\delta)t^\alpha}{8}\sec h^2\left(\frac{\sqrt{2}x}{4}\right) - \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\frac{(1-2\delta)^2}{16}\tan h\left(\frac{\sqrt{2}x}{4}\right)\sec h^2\left(\frac{\sqrt{2}x}{4}\right) + \cdots.\right) \quad \alpha = 1 \qquad (1.4)$$

$$U(x,t) = \left(\frac{1}{2}+\frac{1}{2}\tan h\left(\frac{\sqrt{2}x}{4}\right) + \frac{(1-2\delta)t}{8}\sec^2\left(\frac{\sqrt{2}x}{4}\right) - \frac{t^2(1-2\delta)^2}{32}\tan h\left(\frac{\sqrt{2}x}{4}\right)\sec^2\left(\frac{\sqrt{2}x}{4}\right)\ldots \qquad (1.5)$$

FRDTM's output is exactly in line with the correct answer. The suggested method is tested by comparing the estimated result with the precise result to see whether it is effective and accurate.

Table 1.1: FRDTM's precise solution for the fractional Fitzhugh-Naguma equation and the absolute inaccuracy of the II approximation solution

t_i/x_i	0.1	0.2	0.3
0.1	4.8455×10^{-5}	4.8455×10^{-4}	4.8455×10^{-3}
0.2	3.3554×10^{-5}	3.3554×10^{-4}	3.3554×10^{-3}
0.3	2.7872×10^{-5}	2.7872×10^{-4}	2.7872×10^{-3}

Table 1.2: The absolute difference between the ninth estimated numerical solution via ADM and the actual answer

t_i/x_i	0.1	0.2	0.3
0.1	4.8455×10^{-5}	4.8455×10^{-4}	4.8455×10^{-3}
0.2	3.3554×10^{-5}	3.3554×10^{-4}	3.3554×10^{-3}
0.3	2.7872×10^{-5}	2.7872×10^{-4}	2.7872×10^{-3}

Table 1.3: The absolute difference between the fifth estimated numerical solution via HPM and the actual answer

t_i/x_i	0.1	0.2	0.3
0.1	4.0710×10^{-17}	2.0911×10^{-14}	8.0606×10^{-13}
0.2	3.7487×10^{-17}	1.9339×10^{-14}	7.4868×10^{-13}
0.3	3.2283×10^{-17}	1.6742×10^{-14}	6.5147×10^{-13}

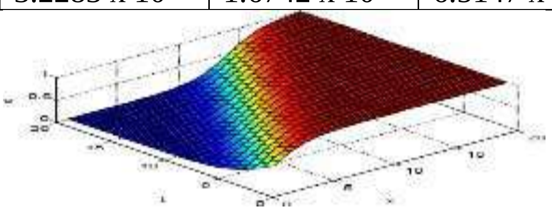


Figure 1.1: Fitzhugh-Nagumo equation phase diagram of u for the precise solution

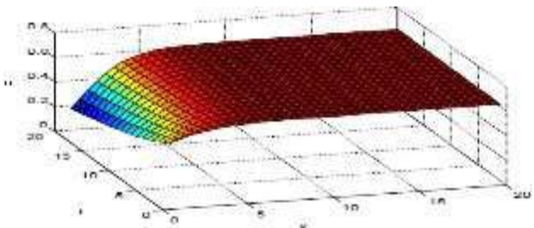


Figure 1.2: Phase plot of u at order of derivative 0.50

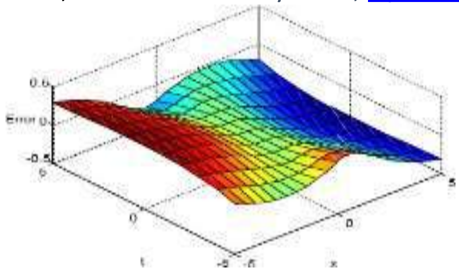
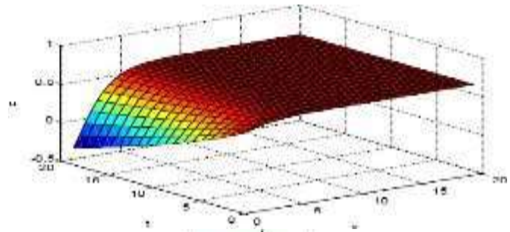


Figure 5.3: Phase plot of u at order of derivative 1.0

Figure 1.4: Phase plot of u at order of derivative 0.75



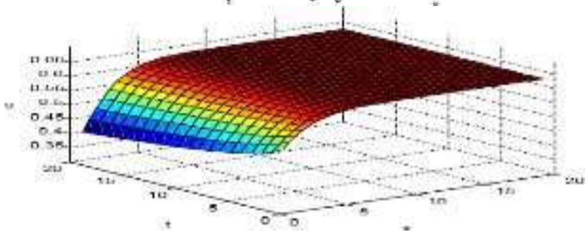


Figure 1.5: Phase plot of u at order derivative 0.25

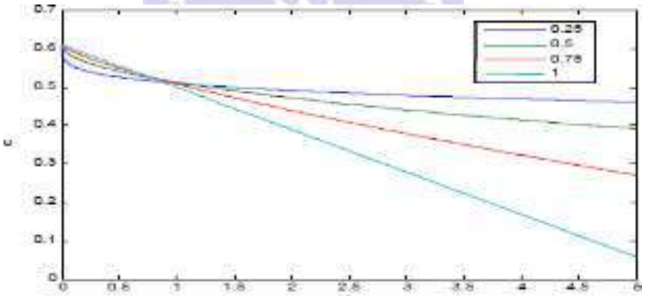


Figure 1.6: Graph of u at x=1 for various order of derivatives

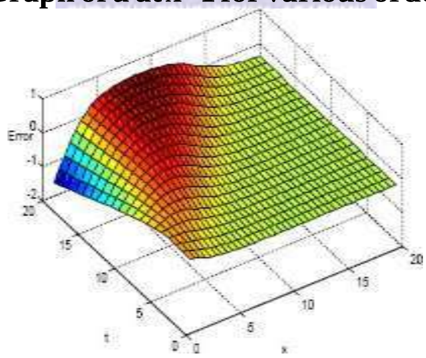


Figure 1.7: Fitzhugh-Nagumo equation approximation error phase diagram at order of derivative 1.0

A simple and effective approach for solving the Fitzhugh-Nauma equation is shown in this section. As can be seen from the comparison with previous work, the presented solutions for =1 produce efficient approximations to the precise answer only after a few iterations. These solutions are identical to those provided by Mehdi, Jalil, and Abbas. In addition, the FRDTM computation is straightforward and easy to understand.

3 FRDTM FOR THE TIME FRACTIONAL ROSENAU-HYMAN EQUATION (FRH)

Using FRDTM, an analytic approximation solution to the time fractional Rosenau-Hyman issue is examined in this section. The Caputo-style fractional differentiations are used. When FRDTM's explanation is compared to the precise answers, it is discovered that the produced findings are quite close to the exact solution in agreement. Then we conduct a thorough study of the FRDTM data up to the second approximation and estimate the inaccuracy. A comparison table of various numerical solutions shows that the current technique provides a reliable, efficient, and convergent solution in the form of an easily computable and convergent series. When liquid drop patterns form, the Rosenau-Hyman time fractional equation (FRH) comes into play. Methods used by Molliq and Noorani to solve a fractional Rosenau-Hyman equation include the use of VIM and HPM techniques.

$$D_t^{\alpha} u = uD_{xxx}(u) + uD_x(u) + 3D_x(u)D_{xx}(u)...,$$

$$u(x,0) = -(8/3)c\cos^2(x/4),$$

with preliminary condition (1.6)

Solution by FRDTM,

Applying FRTDM on equation (5.6), we find the given upswing relation

$$U_{K+1}(x) = \frac{\Gamma(1+\alpha k)}{\Gamma(\alpha+\alpha k+1)}\Bigg[\sum_{r=0}^{k} U_r(x)\frac{\partial^3}{\partial x^3}U_{k-r}(x) + \sum_{r=0}^{k} U_r(x)\frac{\partial}{\partial x}U_{k-r}(x)$$

$$+3\sum_{r=0}^{k} \frac{\partial}{\partial x}U_r(x)\frac{\partial^2}{\partial x^2}U_{k-r}(x)\Bigg],$$

with initial conditions

Using recurrence $U_0(x) = -\frac{8}{3}c\cos^2\frac{x}{4}$ ·al condition (5.8), we get (1.8)

For k = 0

$$U_1(x) = \frac{1}{\Gamma(\alpha+1)}\left(U_0(x)\frac{\partial^3}{\partial x^3}U_0(x) + U_0(x)\frac{\partial}{\partial x}U_0(x) + 3\frac{\partial}{\partial x}U_0(x)\frac{\partial^2}{\partial x^2}U_0(x)\right),$$

$$U_1(x) = \frac{1}{\Gamma(\alpha+1)}\left[-\frac{2}{3}c^2 \sin\frac{x}{2}\right].$$

For $k = 1$, we get

$$U_2(x) = \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}\left[\sum_{r=0}^{1} U_r(x)\frac{\partial^3}{\partial x^3}U_{1-r}(x) + \sum_{r=0}^{1} U_r(x)\frac{\partial}{\partial x}U_{1-r}(x)\right.$$

$$\left.+3\sum_{r=0}^{1}\frac{\partial}{\partial x}U_r(x)\frac{\partial^2}{\partial x^2}U_{1-r}(x)\right]$$

$$U_2(x) = \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}\left(U_0(x)\frac{\partial^3}{\partial x^3}U_1(x) + U_1(x)\frac{\partial^3}{\partial x^3}U_0(x) + U_0(x)\frac{\partial}{\partial x}U_1(x)\right.$$

$$\left.+U_1(x)\frac{\partial}{\partial x}U_0(x) + 3\frac{\partial}{\partial x}U_0(x)\frac{\partial^2}{\partial x^2}U_1(x) + 3\frac{\partial}{\partial x}U_1(x)\frac{\partial^2}{\partial x^2}U_0(x)\right)$$

$$U_2(x) = \frac{c^3}{3\Gamma(2\alpha+1)}\cos\frac{x}{2}.$$

Also so on

$$U_3(x) = \frac{c^4}{6\Gamma(3\alpha+1)}\sin\frac{x}{2},$$

$$U_4(x) = -\frac{c^5}{12\Gamma(4\alpha+1)}\cos\frac{x}{2}$$

Finally the approximate solution of problem (5.6) is found as

$$u(x,t) = \left(-\frac{8}{3}c\cos^2\frac{x}{4} - \frac{2c^2t^{\alpha}}{3\Gamma(\alpha+1)}\sin\frac{x}{2} + \frac{c^3t^{2\alpha}}{3\Gamma(2\alpha+1)}\cos\frac{x}{2} + \frac{c^4t^{3\alpha}}{6\Gamma(3\alpha+1)}\sin\frac{x}{2}\right.$$

$$\left.-\frac{c^5t^{4\alpha}}{12\Gamma(4\alpha+1)}\cos\frac{x}{2}.\right) \quad (1.9)$$

For $\alpha = 1$, we get

$$u(x,t) = -\frac{4c}{3} - \frac{4c}{3}cos\,\frac{x}{2} - \frac{2c^2}{3}tsin\,\frac{x}{2} + \frac{c^3}{6}t^2cos\,\frac{x}{2} + \frac{c^4}{36}t^3sin\,\frac{x}{2} - \frac{c^5}{(12)}\frac{t^4}{(24)}cos\,\frac{x}{2} + \cdots,$$

$$u(x,t) = -\frac{4c}{3} - \frac{4c}{3}cos\,\frac{x}{2}\left(1 - \frac{(ct/2)^2}{2!} + \frac{(ct/2)^4}{4!}\ldots\right) - \frac{4c}{3}sin\,\frac{x}{2}\left(\left(\frac{ct}{2}\right) - \frac{(ct/2)^3}{3!} + \cdots\right),$$

$$= -\frac{4c}{3}\left[1 + cos\frac{x}{2}cos\frac{ct}{2} + sin\frac{x}{2}sin\frac{ct}{2}\right] = -\frac{4c}{3}\left[1 + cos\left(\frac{x-ct}{2}\right)\right] \quad (1.10)$$

which is the exact solution

Table 1.4: Fractional Rosenau-II Hyman's approximation solution error analysis α =1and c =1

X	T	II-Approx. result by FRTDM	Exact	Absolute error $\|U_{Exact} - U_{FRDTM}\|$ at $\alpha = 1$
$\frac{\pi}{4}$	0.2	-2.61	-2.61	0.0
	0.4	-2.6426	-2.642	0.0006
	0.6	-2.6628	-2.6609	0.0019
$\frac{\pi}{2}$	0.2	-2.3657	-2.3656	0.0001
	0.4	-2.4458	-2.4447	0.0011
	0.6	-2.5166	-2.5127	0.0039
	0.2	-1.9642	-1.964	0.0002
$\frac{3\pi}{4}$	0.4	-2.0797	-2.0781	0.0016
	0.6	-2.1902	-2.1848	0.0054
π	0.2	-1.4667	-1.4664	0.0003
	0.4	-1.6	-1.5982	0.0018
	0.6	-1.7333	-1.7274	0.0059

Table 1.5: Fifth term solution through VIM and HPM when α =1

X	T	VIM	HPM
$\frac{\pi}{4}$	0.2	-2.6099	-2.6099
	0.6	-2.6609	-2.6609
	1.0	-2.659	-2.6589
$\frac{\pi}{2}$	0.2	-2.3655	-2.3655
	0.6	-2.5126	-2.5126
	1.0	-2.6127	-2.6127
$\frac{3\pi}{4}$	0.2	-0.4893	-0.4893
	0.6	-0.71125	-0.71125
	1.0	-0.9579	-0.9579
π	0.2	-1.4664	-1.4664
	0.6	-1.7273	-1.7273
	1.0	-1.9725	-1.9725

4 NUMERICAL RESULTS AND DISCUSSIONS

Table 1.4 shows the comparison between the FRDTM results produced at the second approximation and the precise solution for =1 for various values of x and t. As shown in Table 1.5, for various x, t values, the approximate solution using 5th iterations of VIM and HPM for = 1 may be found. The FRDTM is used to solve the Caputo time fractional order Rosenau-Hyman issue that arises while creating liquid droplets. FRH equation with an initial condition has a suggested solution in the form of power series, which does not need discretization, perturbation or He's polynomials. Second approximation results in a great agreement with the fifth term solutions of VIM and HPM. Convergence times for the approach are much quicker than for the VIM and HPM, which are used as an approximation for the technique. Nonlinear fractional derivative problems exist in many fields of practical mathematics, hence semi-analytical techniques are more effective and efficient.

Fractional Order Bio-Mathematical Model for the Darwinism of the Smoking Habit

In this section, we'll employ FRDTM to regularly get the infected person's lucidity needed for the growth of population smoking behaviours. The growth of socially unacceptable behaviors, such as smoking, obesity epidemics, alcohol and cocaine addiction, has repeatedly been linked to the etiology of non-fatal disease in a community. In the current study, we investigate how smoking habits might spread. With the parameter beginning values, we will utilize actual, real-world data. The following presumptions will be taken into account:

1. A regular population is fix that is the birth and death rates are equal but not equal to zero;
2. The total number of individual is unique, but is constantly renewed.
3. Non-smokers are those who have never smoked, regular smokers are those who smoke less than 20 cigarettes per day, heavy smokers are those who smoke more than 20 cigarettes per day, and ex-smokers are those who have previously smoked. They are represented by X, Y, S, and B, respectively. The result is a fractional mathematical model for the development of a smoking habit.

$$D_{\varepsilon}^{\alpha}x(\varepsilon)=\vartheta-(d_0+\vartheta)x(\varepsilon)+d_0x^2(\varepsilon)+\left(d_f-\beta\right)x(\varepsilon)\left(y(\varepsilon)+s(\varepsilon)\right)+\left(\frac{d_0+d_f}{2}\right)x(\varepsilon)b(\varepsilon), \quad (1.11)$$

$$D_{\varepsilon}^{\alpha}y(\varepsilon)=\beta x(\varepsilon)(y(\varepsilon)+s(\varepsilon))+\rho b(\varepsilon)+\alpha s(\varepsilon)-\left(\gamma+\lambda+\vartheta+d_f\right)y(\varepsilon)+d_0x(\varepsilon)y(\varepsilon)$$
$$+d_fy(\varepsilon)\left(y(\varepsilon)+s(\varepsilon)\right)+\left(\frac{d_0+d_f}{2}\right)y(\varepsilon)b(\varepsilon), \quad (1.12)$$

$$D_{\varepsilon}^{\alpha}s(\varepsilon)=\gamma y(\varepsilon)-\left(\alpha+\delta+\vartheta+d_f\right)s(\varepsilon)+d_0x(\varepsilon)s(\varepsilon)+d_fs(\varepsilon)\left(y(\varepsilon)+s(\varepsilon)\right)+\left(\frac{d_0+d_f}{2}\right)s(\varepsilon)b(\varepsilon), \quad (1.13)$$

$$D_{\varepsilon}^{\alpha}b(\varepsilon)=\lambda y(\varepsilon)+\delta s(\varepsilon)-\left(\rho+\vartheta+\frac{d_0+d_f}{2}\right)b(\varepsilon)+d_0x(\varepsilon)b(\varepsilon)+d_fb(\varepsilon)\left(y(\varepsilon)+s(\varepsilon)\right)$$
$$+\left(\frac{d_0+d_f}{2}\right)b^2(\varepsilon) \quad (1.14)$$

$x=\frac{X}{P}$, $y=\frac{Y}{P}$, $s=\frac{S}{P}$, $b=\frac{B}{P}$ where P shows the total fix population.

Solution by FRDTM,

By using FRDTM in equation (5.11)

$$X_{k+1}(\varepsilon)=\frac{\Gamma(\alpha k+1)}{\Gamma(\alpha+1+\alpha k)}\left[\vartheta\delta(k)-(d_0+\vartheta)X_k(\varepsilon)+d_0\sum_{\gamma=0}^{k}X_\gamma(\varepsilon)X_{k-\gamma}(\varepsilon)+\left(d_f-\beta\right)\sum_{\gamma=0}^{k}X_\gamma(\varepsilon)Y_{k-\gamma}(\varepsilon)\right.$$

where $\vartheta=0.01, d_0=0.0087, d_f=0.0132, \beta=0.0381, \rho=0.0425, \alpha=0.1244$
$\gamma=0.1175$, $\lambda=0.0498$, $\delta=0.0498$

with preliminary condition

X(0) = 0.5045 , S(O)= 0.1559 , Y(O) = 0.2059 , B(O)= 0,1337

We get

$$X_1(\varepsilon)=\frac{1}{\Gamma(1+\alpha)}\left[\vartheta-(d_0+\vartheta)(0.5045)+d_0(0.5045)^2+\left(d_f-\beta\right)(0.5045)(0.2059)\right.$$
$$\left.+\left(d_f-\beta\right)(0.5045)(0.1559)+\left(\frac{d_0+d_f}{2}\right)(0.5045)(0.1337)\right]$$

$$X_1(\varepsilon)=\frac{1}{\Gamma(1+\alpha)}[(0.01)-(0.0087+0.01)0.5045+0.0087(0.5045)^2$$
$$+(0.0132-0.0381)(0.5045)(0.2059)+(0.0132-0.0381)(0.5045)(0.1559)$$
$$\left.+\left(\frac{0.0087+0.0132}{2}\right)(0.5045)(0.1337)\right]$$

$$X_1(\varepsilon) = \frac{1}{\Gamma(1+\alpha)}[(0.01) - (0.0094) + (0.0022) - (0.0026) - (0.0019) + (0.0007)]$$

$$X_1(\varepsilon) = \frac{1}{\Gamma(1+\alpha)}[(-0.001)] \tag{1.16}$$

$$Y_{k+1}(\varepsilon) = \frac{\Gamma(1+\alpha k)}{\Gamma(\alpha+1+\alpha k)}\left[\sum_{\gamma=0}^{k} \beta X_y(\varepsilon)Y_{k-\gamma}(\varepsilon) + \sum_{y=0}^{k} \beta X_\gamma(\varepsilon)S_{1-\gamma}(\varepsilon) + \rho B_k(\varepsilon) + \alpha S_k(\varepsilon)\right.$$

$$-(\gamma+\lambda+\vartheta+d_f)Y_k(\varepsilon) + d_0\sum_{\gamma=0}^{k} X_y(\varepsilon)Y_{k-\gamma}(\varepsilon) + d_f\sum_{\gamma=0}^{k} Y_\gamma(\varepsilon)Y_{k-\gamma}(\varepsilon)$$

$$\left. +d_f\sum_{\gamma=0}^{k} Y_\gamma(\varepsilon)S_{k-\gamma}(\varepsilon) + \left(\frac{d_0+d_f}{2}\right)\sum_{\gamma=0}^{k} Y_\gamma(\varepsilon)B_{k-\gamma}(\varepsilon)\right] \tag{1.17}$$

For k = 0

$$Y_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}\left[\beta X_0Y_0 + \beta X_0S_0 + \rho B_0 + \alpha S_0 - (\gamma+\lambda+\vartheta+d_f)Y_0 + d_0(X_0Y_0)\right.$$

$$\left. +d_f(Y_0Y_0) + d_f(Y_0S_0) + \left(\frac{d_0+d_f}{2}\right)Y_0B_0\right]$$

$$Y_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[(0.0381)(0.5045)(0.2059) + (0.0381)(0.5045)(0.1559) + (0.0425)(0.1337)$$

$$+(0.1244)(0.1559) - (0.1175 + 0.0498 + 0.01 + 0.0132)(0.2059)$$

$$+(0.0087)(0.05045)(0.2059) + (0.0132)(0.2059)^2 + (0.0132)(0.2059)(0.1559)$$

$$\left. + \left(\frac{0.0087+0.0132}{2}\right)(0.2059)(0.1337)\right]$$

$$Y_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[0.0039 + 0.0029 + 0.056 + 0.0193 - 0.039 + 0.0009 + 0.0005 + 0.0004 + 0.0003]$$

$$Y_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[-0.0052] \tag{1.18}$$

Using FRDTM in equation (5.13)

$$S_{k+1} = \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha k+\alpha+1)}\left[\gamma Y_k(\varepsilon) - (\alpha+\delta+\vartheta+d_f)S_k(\varepsilon) + d_0\sum_{\gamma=0}^{k} X_\gamma(\varepsilon)S_{k-\gamma}(\varepsilon)\right.$$

$$\left. +d_f\sum_{\gamma-0}^{k} S_\gamma(\varepsilon)Y_{k-\gamma}(\varepsilon) + d_f\sum_{\gamma-0}^{k} S_\gamma(\varepsilon)S_{k-\gamma}(\varepsilon) + \left(\frac{d_f+d_0}{2}\right)\sum_{\gamma-0}^{k} S_\gamma(\varepsilon)B_{k-\gamma}(\varepsilon)\right] \tag{1.19}$$

For $k = 0$

$$S_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}\left[\gamma Y_0(\varepsilon) - (\alpha+\delta+\vartheta+d_f)S_0(\varepsilon) + d_0(X_0(\varepsilon)S_0(\varepsilon)) + d_f(S_0(\varepsilon)Y_0(\varepsilon))\right.$$

$$\left. +d_f(S_0(\varepsilon)S_0(\varepsilon)) + \left(\frac{d_0+d_f}{2}\right)(S_0(\varepsilon)B_0(\varepsilon))\right],$$

$$S_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[(0.1175)(0.2059) - (0.1244 + 0.0498 + 0.01 + 0.0132)(0.1559)$$

$$+(0.0087)(0.5045)(0.1559) + (0.0132)(0.2059)(0.1559)$$

$$\left. +(0.0132)(0.1559)^2 + \left(\frac{0.0087+0.0132}{2}\right)(0.1559)(0.1337)\right],$$

$$S_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[0.0241 - 0.0307 + 0.0006 + 0.0004 + 0.0003 + 0.0002],$$

$$S_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}(-0.0051) \tag{1.20}$$

Using FRDTM in equation (5.14)

$$B_{k+1} = \frac{\Gamma(1+\alpha k)}{\Gamma(\alpha k+\alpha+1)}\left[\lambda Y_k(\varepsilon) + \delta S_k(\varepsilon) - \left(\rho+\vartheta+\frac{d_0+d_f}{2}\right)B_k(\varepsilon) + d_0\sum_{\gamma-0}^{k} X_\gamma(\varepsilon)B_{k-y}(\varepsilon)\right.$$

$$\left. +d_f\sum_{\gamma=0}^{k} B_\gamma(\varepsilon)Y_{k-\gamma}(\varepsilon) + d_f\sum_{\gamma=0}^{k} B_\gamma(\varepsilon)S_{k-\gamma}(\varepsilon) + \left(\frac{d_f+d_0}{2}\right)\sum_{\gamma-0}^{k} B_\gamma(\varepsilon)B_{k-\gamma}(\varepsilon)\right].$$

For k = 0

$$B_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}\left[\lambda Y_0(\varepsilon) + \delta S_0(\varepsilon) - \left(\rho + \vartheta + \frac{d_0+d_f}{2}\right)B_0(\varepsilon) + d_0(X_0(\varepsilon)B_0(\varepsilon))\right.$$

$$\left. + d_f(B_0(\varepsilon)Y_0(\varepsilon)) + d_f(B_0(\varepsilon)S_0(\varepsilon)) + \left(\frac{d_0+d_f}{2}\right)(B_0(\varepsilon)B_0(\varepsilon))\right]$$

$$B_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[(0.0498)(0.02059) + (0.0498)(0.1559)$$

$$-\left(0.0425 + 0.01 + \left(\frac{0.0087+0.0132}{2}\right)\right)(0.1337) + (0.0087)(0.5045)(0.1337)$$

$$+(0.0132)(0.1337)(0.2059) + (0.0132)(0.1337)(0.1559) + \left(\frac{0.0087+0.0132}{2}\right)(0.1337)^2\Big]$$

$$B_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}[0.01025 + 0.00776 - 0.00848 + 0.00058 + 0.00036 + 0.00027 + 0.00019]$$

$$B_1(\varepsilon) = \frac{1}{\Gamma(\alpha+1)}(0.01093) \qquad (1.21)$$

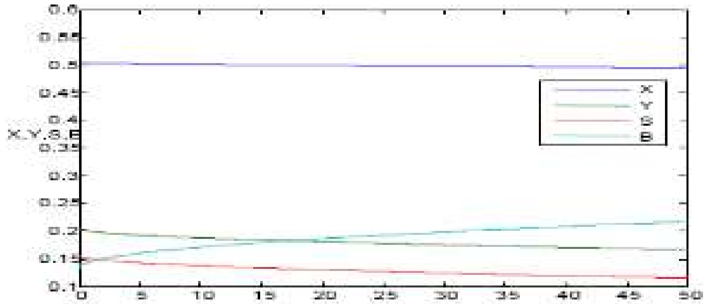


Figure 1.9: Plot of X, Y, S, B vs. time at order of derivative 0.50

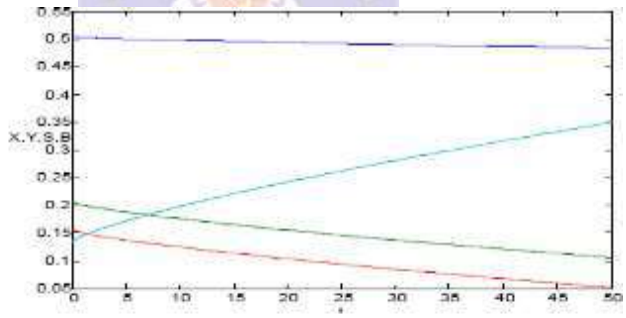


Figure 1.10: Plot of X, Y, S, B vs. time at order of derivative 0.75

The fractional derivative equation approach, which includes a specific mathematical formula for smoking habit development in the present population, is used to derive approximations of solutions in the section above. Accuracy, overall productivity, and dependability are readily shown in graphic charts of absolute errors and approximations of answers.

5. CONCLUSION

The primary advantage of the suggested numerical approach is its capacity to provide much superior data in terms of the regular average solution for a specific time span. The creation of efficient binary schemes for nonlinear fractional ordinary differential equations and their application to solve mathematical models will help to clarify, quantify, and improve the aforementioned research goals.

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