

# Hybrid Type Demand Inventory Model for Perishable Items with Advance Payment Concerned Discount Facilities under the Effect of Inflation and Preservation Technology

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## Abstract:

The aim of this research is to demonstrate the optimal strategy of inventory system for perishable items with hybrid type demand under the effect of inflation partial backlogging and preservation technology with a certain ratio. Preservation technology (PT) used to control the deterioration. In this discipline, the concept of advance payment policy with discount facility is includes. This inventory is modeled mathematically by governing differential equations, and solving all these equation, in that behalf average benefit of introduced model is acquired as nonlinear maximization problem. To examine an optimality of the solution of the yield nonlinear maximization problem, some results are described with help of concavity and optimal conditions of objective function of the non-linear maximization problem. Sensitive analysis and graphical representation solved by MATHEMATICA SOFTWARE 7.0.

**Keywords:** Hybrid demand, inflation, preservation technology, perishable items

## 1. Introduction:

We know that deterioration is natural process. Deterioration means decaying, spoliation, vaporization etc. but inventory is major issue we can't avert inventory. Now a days business policy dependent on competitive marketing strategies. It is very much challenging for each businessman or seller to outlive in competition. In the market a new trend become discount due to the pre payment policy. There are many researchers, academicians and scholars are very interested to study the inventory models with under the effect of inflation and pre payment policy. There are many researchers are work done on discount facility under the pre payment policy. Some of those are described here with short description.

Das et al. (2020) considered a deteriorated inventory problem of items with preservation facility. And demand is dependent on selling price. And also they allowed partially backlogged shortages. Harit and Singh (2020) incorporated the concept of conservation techniques and warehousing. And they used time dependent demand function.

Khakzad and Gholamain (2020) developed an inventory model for deteriorating goods with advance payment policy. In this model, average deterioration rate was related to the number of inspections at every period. Namdeo et al. (2020) presented an inventory model with constant rate of deterioration of the items. And they allowed shortage. And demand dependent on stock and price sensitive.

Nasir and Vo (2020) studied the implication of inflation target for the exchange rate pass thru trade balance and inflation. Sundarajan et al. (2020) have analyzed inventory model in which demand determined by price and time. They allowed shortage and partially backlogged. Charles and Marie (2021) used hyperinflation concept over many decade and accompanied by a weakling of the domestic currency which was particularly salient. Das et al. (2021) studied a model for deteriorating items with preservation technology. And demand is dependent on selling price. In this paper they allowed shortage.

Advance payment is other most popular business policy in the business management area. Sometimes it happen that demand exponential increase for a product and insufficient supply, the stock level of that particular items is volatized in market. To manage this type cases, the purchaser aspire to prepayment either full purchasing cost or a constant percentage of full purchasing cost to make sure the guarantee of the on time delivery of that goods. Many researchers and scholars work done on advance payment policy.

Duary et al.(2021) constructed inventory model for deteriorating items and demand is dependent on selling price, time and frequency of advertisement. And they allowed partially backlogging shortage. Ghosh et al. (2021) investigated an EOQ (economic order quantity)

model for perishable items under advance and delayed payment policies. And demand was dependent on time. Halim et al. (2021) discussed a production inventory model for fragment quality items. And demand dependent on stock and price.

Manna et al. (2021) developed a model for imperfect and perfect items. They considered partially pre payment policy. Mashud et al. (2021) have established a model for non-instantaneous deteriorating items. This paper considered advance payment policy, carbon emission, GT(green technology) and PT(preservation technology) and they allowed shortage. Kumar (2021) elaborated model for decaying items under the effect of preservation of items. And whole study carried out under the effect of inflationary environment.

On other hand, it is persuaded that the optimal policy of inventory model is miserably impacted by irresponsible deterioration. Thus, to reduce irresponsible deterioration technologists and many other engineers have developed a technology which is called preservation technology. On the preservation technology many academicians and researchers worked out.

Rahman et al. (2021) developed economic order model (EOQ) with advance payment policy and preservation technology. They allowed shortage and demand is hybrid price and stock dependent. Rathore and Sharma (2021) presented a preservation technology model for deteriorating items with constant rate of deterioration. Saha et al. (2021) formulated an inventory model for non-instantaneous deteriorating items. They used preservation technology to reduce the deterioration. And demand type was quadratic time dependent.

Sepehri et al. (2021) elaborated a sustainable production inventory model for poor quality deteriorating items. And deterioration rate is constant. They used preservation technology to control deterioration. Sundarajan et al. (2021) analyzed an EOQ (economic order quantity) model for non-instantaneous deteriorating items with the effect of inflation. And demand function dependent on selling price. Bhawaria and Rathore (2020) have developed a production inventory model for deteriorating items and specially focused on effect of inflation and preservation technology (PT). And they allowed shortage and used demand function is hybrid type. Roy et al. (2021) studied a model for deteriorating items and advance payment policy. And they used preservation technology.

We developed this EOQ model based on some special facts which are inflation, preservation technology, hybrid demand concept, discount and advance payment policy with partial backlogging shortage and the proposed model is formulated mathematically by using ordinary differential equations and the relevant optimization is yield as a profit maximization problem. The optimization problem is solved analytically to check the authenticity of the model, numerical example and sensitive analysis solved by Mathematica software 7.0. The remaining study is well thought-out as follows.

The remaining study is well thought out as follows:

In Section 2 assumptions and notations are discussed. Section 3 discussed the detail of formulation inventory model with associative costs. Section 4 presents theoretical derivation. Section 5 Numerical example. In section 6 sensitive analyses. And finally in section 7 conclusion and future scope of the research.

## 2. Notation and Assumptions

### 2.1 Notation

We used the following symbols throughout the paper development.

Notations	Units	Description
$I(t)$	units	inventory level at any time $t$ , where $0 \leq t \leq T$
$C_o$	₹/order	Per order replenishment cost
$C_p$	₹/unit	Holding cost per unit per time unit
$C_L$	₹/unit/unit time	Opportunity cost per unit per time unit
$C_D$	₹/unit/unit time	Deterioration cost per unit time
$C_B$	₹/unit/unit time	Shortage cost per unit per time unit
$\omega$	%	Backlogging rate
$\tau_\theta$	$0 < \theta < 1$	Deterioration rate, where $\tau_\theta = (\theta - m(\xi))$ ,

		$m(\xi) = e^{-b\xi}$
$d_i$	%	Discount percent based on advance payment
$s$	units	Max. Product amount at very starting of a cycle
$R_{ms}$	units	Max. Shortage amount at the end of a cycle
$Q$	units	Lot size of total order
$\xi$	₹/unit time	preservation cost
$TP(T, t_1)$	₹/unit time	total benefit per unit time
$r$		inflation rate
$p$		selling price
<b>Decision variables</b>		
$T$	unit time	cycle length
$t_1$	unit time	time at which inventory level becomes zero

## 2.2 Assumptions:

- The hybrid type demand function  $f(p, I(t)) = (D(p) + aI_i(t))$ , : where  $a > 0$ , depends on selling price ( $p$ ) of the production at time  $t$ .  $D(p) = \tau(x_1 - yp) + (1 - \tau)x_2p^{-\gamma}$  Where;  $0 \leq t \leq 1, x_1 > 0, x_2 > 0, y > 0, \frac{x_1}{y} \geq p$  and  $y > 1$
- The deterioration is started immediately after getting the lot and its rate is constant  $\theta (0 < \theta < 1)$ .
- The inventory planning horizon is infinite.
- Shortages are allowed but fulfilled with fixed ratio  $\omega$ .
- Lead time is negligible and replenishment rate is limitless.
- To reduce the deterioration rate of items, preservation technology (PT) has been contained and new deterioration rate is taken as  $\theta = e^{-b\xi}$ , where  $b$  is controlling parameter of preservation technology investment.

## 3. Mathematical model formulation:

The businessman places order of  $s + R_{ms}$  units of deteriorating items by paying the whole purchase price at a time period to  $L$  unit time from receiving moment and then, in return to advance payment. When the businessmen gain the goods, he fulfils the whole backlogged items immediately and rest stock decreases due to demand and deterioration. And at time  $t=t_1$ , inventory level becomes zero. Instantly, shortages arise and these are partially backlogged shortages with rate  $\omega$ . So, inventory system is introduced with the help of the following differential equations:

$$\frac{dI_1(t)}{dt} + \tau_\theta I_1(t) = -D(p) - \delta I_1(t); \quad 0 < t \leq t_1 \quad (1)$$

$$\frac{dI_2(t)}{dt} = -\omega D(p); \quad t_1 < t \leq T \quad (2)$$

With the boundary conditions  $I(0)=s, I(t_1)=0, I(T)=-R_{ms}$  and  $I(t)$  is continuous at  $t=t_1$ .

Solving the eq. 1 with the help of condition  $I(t_1)=0$ .

$$I_1(t) = \frac{D(p)}{\tau_\theta + \delta} [e^{(\tau_\theta + \delta)(t_1 - t)} - 1] \quad (3)$$

Again, soaking the condition  $I(0)=s$ .

$$s = \frac{D(p)}{\tau_\theta + \delta} [e^{(\tau_\theta + \delta)(t_1)} - 1] \quad (4)$$

Solving eq. (2) with the boundary condition  $I(t_1)=0$ .

$$I_2(t) = D(p)\omega(t_1 - t) \quad (5)$$

Exploiting the auxiliary condition  $I(T) = -R_{ms}$ , one can derive.

$$R_{ms} = \omega D(p)(T - t_1) \quad (6)$$

Total number of ordered amount is-

$$Q = (R_{ms} + s) \quad (7)$$

$$Q = D(p) \left[ \frac{1}{(\tau_\theta + \delta)} (e^{(\tau_\theta + \delta)(t_1)} - 1) + \omega(T - t_1) \right] \quad (8)$$

The entire inventory benefit consists of the following costs.

### Ordering cost is

$$C_o \text{ per cycle} \quad (9)$$

### Holding cost

$$C_{(H)} = C_{(H)} \int_0^{t_1} e^{-rt} I_1(t) dt$$

$$C_{(H)} = \frac{D(p)C_H}{(\tau_\theta + \delta)} \left[ \frac{1}{r} (e^{-r(t_1)} - 1) - \frac{1}{(r + \tau_\theta + \delta)} (-e^{-r(t_1)} + e^{(\tau_\theta + \delta)(t_1)}) \right] \quad (10)$$

#### Purchasing Cost

$$C_{(p)} = (1 - d_i)C_p(s + R_{ms})$$

$$C_{(p)} = (1 - d_i)C_p \left[ D(p) \left\{ \frac{1}{(\tau_\theta + \delta)} (e^{(\tau_\theta + \delta)(t_1)} - 1) + \omega(T - t_1) \right\} \right] \quad (11)$$

#### Capital cost

$$C_C = LI_e(1 - d_i)C_p \left[ D(p) \left\{ \frac{1}{(\tau_\theta + \delta)} (e^{(\tau_\theta + \delta)(t_1)} - 1) + \omega(T - t_1) \right\} \right] \quad (12)$$

#### Shortage Cost

$$S_C = -C_B \int_{t_1}^T e^{-rt} I_2(t) dt$$

$$S_C = C_B D(p) \omega \left[ \frac{t_1}{r} (e^{-r(t_1)} - e^{-r(T)}) + \left( \frac{1}{r} (Te^{-r(T)} - t_1 e^{-r(t_1)}) - \frac{1}{r^2} (e^{-r(T)} + e^{-r(t_1)}) \right) \right] \quad (13)$$

#### Opportunity Cost

$$O_C = C_L \int_{t_1}^T (1 - \omega) D(p) e^{-rt} dt$$

$$O_C = \frac{D(p)(1 - \omega)C_L}{r} (e^{-rT} - e^{-rt_1}) \quad (14)$$

#### Sales revenue

$$S_R = p(D(p)t_1 + R_{ms}) \quad (15)$$

$$S_R = p(D(p)t_1 + \omega D(p)(T - t_1)) \quad (16)$$

#### Preservation technology cost

$$PTC = \xi T \quad (17)$$

The total benefit of the system is given by-

$$X = S_R - O_C - S_C - C_C - C_{(H)} - C_0 - (1 - d_i)C_p(s + \omega D(p)(T - t_1)) - PTC$$

$$X = p(D(p)t_1 + \omega D(p)(T - t_1)) - \frac{D(p)(1 - \omega)C_L}{r} (e^{-rT} - e^{-rt_1}) - C_B D(p) \omega \left[ \frac{t_1}{r} (e^{-r(t_1)} - e^{-r(T)}) + \left( \frac{1}{r} (Te^{-r(T)} - t_1 e^{-r(t_1)}) - \frac{1}{r^2} (e^{-r(T)} + e^{-r(t_1)}) \right) \right] - LI_e(1 - d_i)C_p \left[ D(p) \left\{ \frac{1}{(\tau_\theta + \delta)} (e^{(\tau_\theta + \delta)(t_1)} - 1) + \omega(T - t_1) \right\} \right] - \frac{D(p)C_H}{(\tau_\theta + \delta)} \left[ \frac{1}{r} (e^{-r(t_1)} - 1) - \frac{1}{(r + \tau_\theta + \delta)} (-e^{-r(t_1)} + e^{(\tau_\theta + \delta)(t_1)}) \right] - C_0 - (1 - d_i)C_p \left[ D(p) \left\{ \frac{1}{(\tau_\theta + \delta)} (e^{(\tau_\theta + \delta)(t_1)} - 1) + \omega(T - t_1) \right\} \right] - \xi T \quad (18)$$

Hence, benefit per unit time is given by-

$$TP(t_1, T) = \frac{X}{T} \quad (19)$$

Now from equation (18), we need to find optimal value of the decision variables  $t_1$  and  $T$  as well as whole benefit of the system. In next section we are going to describe the optimality criterion of the optimization problem (18) in description.

## 4. Theoretical derivations

Hither, we have discussed concavity of the benefit (18) by soaking the result of cambini and martin (2009). According to the theorems 3.2.9 & 3.2.10 of cambini and martin (2009), any function of the form.

$$\Gamma t = \frac{G(t)}{H(t)}, \text{ where } t \in R^n \quad (19)$$

Is strictly pseudo concave, If  $G(t)$  isn't only differential but also rigidly concave however,  $H(t)$  is affine and rigidly positive as well. Using result of (19), we can show that the objective function (18) as a dummy concave function.

Moreover, we have shown the concavity of the total benefit  $TP(t_1, T)$  with respect to  $t_1$  and  $T$ .

#### Necessary Conditions for Maximization the Total Benefit $TP(t_1, T)$ .

The first order partial derivatives  $TP(t_1, T)$  with respect to  $t_1$  and  $T$  are equating with zero.

After rearranging the terms, we get.



$$\frac{\partial TP(t_1, T)}{\partial t_1} = \frac{D(p)}{T} \left[ p - \omega + (1 - \omega)C_L e^{-rt_1} + \frac{C_B \omega}{r^2} \{e^{-rt_1}(1 - rt_1)\} - (1 - d_i)C_p(e^{\tau\theta+\delta} - \omega)(1 + LI_e) + \frac{C_H}{\tau\theta+\delta} \{e^{-rt_1} + \frac{(\tau\theta+\delta)e^{(\tau\theta+\delta)t_1}}{\tau\theta+\delta+r}\} \right] = 0 \quad (20)$$

And

$$\frac{\partial TP(t_1, T)}{\partial T} = \frac{D(p)}{T} \left[ \omega + (1 - \omega)C_L e^{-rT} - C_B \omega \{t_1 e^{-rT} + (-T)e^{-rT} - (1 - d_i)C_p \omega (1 + LI_e) - \xi\} - \frac{1}{T} TP(t_1, T) \right] = 0 \quad (21)$$

Solving eq. (20) & (21) simultaneously, we yield the optimal values of  $t_1$  and  $T$ . We can prove that  $TP(t_1, T)$  reaches the global maximum value at  $(t_1^*, T^*)$  by investigating the sufficient condition for maximization of it.

**Theorem.1** If the

$$\delta < \left\{ \frac{r(C_H + 3C_B t_1 \omega - 3t_1 \tau \theta C_H) - 2C_B \omega + r^2(-C_L + C_L \omega - \omega C_L t_1 - C_B \omega t_1^2) + r^3 C_L t_1}{3r C_H t_1} \right\} \quad \text{Hessian matrix for}$$

$TP(t_1, T)$  is always negative definite, and hence  $TP(t_1, T)$  obtains the global maximum at the point  $(t_1^*, T^*)$  and the point  $(t_1^*, T^*)$  is unique.

**Proof:-** In this theorem we can prove by investigating the first principle minor of the Hessian matrix  $TP(t_1, T)$  is negative while the second principal minor is positive. To make the hessian matrix  $TP(t_1, T)$ , second order partial derivatives are featured with respect to  $t_1$  and  $T$ . let us consider

$$G(t_1, T) = X \quad (22)$$

And

$$H(t_1, T) = T \quad (23)$$

Differentiating partially with respect to  $t_1$  of equation (22) we get

$$\frac{\partial G(t_1, T)}{\partial t_1} = D(p) \left[ p - \omega + (1 - \omega)C_L e^{-rt_1} + \frac{C_B \omega}{r^2} \{e^{-rt_1}(1 - rt_1)\} - (1 - d_i)C_p(e^{\tau\theta+\delta} - \omega)(1 + LI_e) + \frac{C_H}{\tau\theta+\delta} \{e^{-rt_1} + \frac{(\tau\theta+\delta)e^{(\tau\theta+\delta)t_1}}{\tau\theta+\delta+r}\} \right] \quad (24)$$

Again, differentiating partially with respect to both  $t_1$  and  $T$  of equation (24) we get.

$$\frac{\partial^2 G(t_1, T)}{\partial t_1^2} = D(p) \left[ -r(1 - \omega)C_L e^{-rt_1} - \frac{C_B \omega e^{-rt_1}}{r} (2 - rt_1) + \frac{C_H}{\tau\theta+\delta} \{-re^{-rt_1} + \frac{(\tau\theta+\delta)^2 e^{(\tau\theta+\delta)t_1}}{\tau\theta+\delta+r}\} \right] \quad (25)$$

And

$$\frac{\partial^2 G(t_1, T)}{\partial t_1 \partial T} = 0 \quad (26)$$

Again, differentiating equation (22) partially with respect to  $T$  we get.

$$\frac{\partial G(t_1, T)}{\partial T} = D(p) \left[ \omega + (1 - \omega)C_L e^{-rT} - C_B \omega \{t_1 e^{-rT} + (-T)e^{-rT} - (1 - d_i)C_p \omega (1 + LI_e) - \xi\} \right] \quad (27)$$

Again differentiating equation (27) partially with respect to  $T$  we obtain.

$$\frac{\partial^2 G(t_1, T)}{\partial T^2} = -D(p) [r(1 - \omega)C_L e^{-rT} - \omega C_B e^{-rT} \{-rt_1 + (1 - rT)\}] \quad (28)$$

The Hessian Matrix of  $G(t_1, T)$  is given by

$$\begin{bmatrix} \frac{\partial^2 G(t_1, T)}{\partial T^2} & \frac{\partial^2 G(t_1, T)}{\partial T \partial t_1} \\ \frac{\partial^2 G(t_1, T)}{\partial t_1 \partial T} & \frac{\partial^2 G(t_1, T)}{\partial t_1^2} \end{bmatrix}. \text{ The first principal minor is}$$

$$\frac{\partial^2 G(t_1, T)}{\partial T^2} = -D(p) [r(1 - \omega)C_L e^{-rT} - \omega C_B e^{-rT} \{-rt_1 + (1 - rT)\}] < 0. \quad \text{The second principal minor is}$$

$$\begin{vmatrix} \frac{\partial^2 G(t_1, T)}{\partial T^2} & \frac{\partial^2 G(t_1, T)}{\partial T \partial t_1} \\ \frac{\partial^2 G(t_1, T)}{\partial t_1 \partial T} & \frac{\partial^2 G(t_1, T)}{\partial t_1^2} \end{vmatrix} = \frac{\partial^2 G(t_1, T)}{\partial T^2} \frac{\partial^2 G(t_1, T)}{\partial t_1^2} - \frac{\partial^2 G(t_1, T)}{\partial t_1 \partial T} \frac{\partial^2 G(t_1, T)}{\partial T \partial t_1}$$

$$= [(D(p))^2 \left[ -r(1-\omega)C_L e^{-rt_1} - \frac{C_B \omega e^{-rt_1}}{r} (2-rt_1) + \frac{C_H}{\tau_\theta + \delta} \left\{ -r e^{-rt_1} + \frac{(\tau_\theta + \delta)^2 e^{(\tau_\theta + \delta)t_1}}{\tau_\theta + \delta + r} \right\} \right] [r(1-\omega)C_L e^{-rT} - \omega C_B e^{-rT} \{-rt_1 + (1-rT)\}] - \delta]$$

It shows that the second principal minor is positive if  $\left\{ \frac{r(C_H + 3C_B t_1 \omega - 3t_1 \tau_\theta C_H) - 2C_B \omega + r^2(-C_L + C_L \omega - \omega C_L t_1 - C_B \omega t_1^2) + r^3 C_L t_1}{3r C_H t_1} \right\} > 0$ . Since  $\frac{\partial^2 G(t_1, T)}{\partial T^2} < 0$  and  $H_{22} > 0$ , function  $G(t_1, T)$  is strictly concave and differentiable.  $G(t_1, T) = T$  is strictly positive and affine function. Now  $TP(t_1, T)$  stores the global maximum value at the unique point which is obtained from the necessary conditions.

## 5. Numerical Example

To justify the consistency of the proposed model, one example is taken and solved by mathematica software. In this example units of the business cycle and stock-in-period are taken in month. Units of starting stock, maximum shortage are taken in quintal.

The values of the inventory parameters for this example are taken as-

$x_2 = 3, y = 5, p = 2, \tau = 2, \theta = 0.002, \gamma = 1, x_1 = 2, C_L = 500, \omega = 0.6, d_i = 0.4, C_p = 15, b = 1, I_e = 7000, L = 2, r = 0.002, \delta = 0.0004, C_o = 8000, C_B = 35$

The optimal values are-

$T^* = 4091.37, \xi^* = 6491.62, t_1^* = 2.78583 \times 10^{12}, (TP)^* = 4.54266 \times 10^6$

Here \* shows the optimal values

## 6. Sensitivity Analysis:

Table 1.

S.NO.	PARAMETERS	CHANGES	$t_1 \times 10^{12}$	T	$\xi$	$TP \times 10^{10}$
1.	$C_B$	25	3.90745	5527.52	6674.53	-8.63719
		35	2.78583	4091.37	6491.62	0.000454266
		45	2.07873	3293.51	6306.90	-16.9902
2.	$r$	0.001	3.05947	4591.37	6480.28	4.86516
		0.002	2.78583	4091.37	6491.62	0.000454266
		0.003	2.62796	3924.70	6496.04	30.4574
3.	$x_2$	2	2.65759	4091.37	6306.08	4.51687
		3	2.78583	4091.37	6491.62	0.000454266
		4	2.81408	4091.37	6677.15	3.96473
4.	$x_1$	1	3.04881	4091.37	7233.76	14.47211
		2	2.78583	4091.37	6491.62	0.000454266
		3	2.42286	4091.37	5749.48	-17.9968
5.	$C_L$	450	2.17813	4091.32	5791.74	39.2696
		500	2.78583	4091.37	6491.64	0.000454266
		550	3.35687	4090.42	7191.49	3.70248
6.	$p$	1	1.41326	4091.32	3346.51	0.0000258867
		2	2.78583	4091.37	6491.62	0.000454266
		3	4.199798	4091.42	9989.80	-13.5837
7.	$\gamma$	0.5	2.83306	4091.37	6722.17	-255.464
		1	2.78583	4091.37	6491.62	0.000454266
		1.5	2.66708	4091.37	6328.59	-2390.66
8.	$\theta$	0.001	2.78583	4091.37	6491.62	0.000454258
		0.002	2.78583	4091.37	6491.62	0.000454266
		0.003	2.78583	4091.37	6491.62	0.000454275

Table 2. The following interpretations are analyzed from above table 1.

S. No	PARAMETERS	CHANGES	$t_1 \times 10^{12}$	T	$\xi$	$TP \times 10^{10}$
1.	$C_B$	↓	↑	↑	↑	↓
		↑	↓	↓	↓	↓

2.	<b>R</b>	↓	↑	*	↓	↑
		↑	↓	↓	↑	↓
3.	<b>x<sub>2</sub></b>	↓	↓	*	↓	↑
		↑	↑	*	↑	↑
4.	<b>x<sub>1</sub></b>	↓	↑	*	↑	↑
		↑	↓	*	↓	↓
5.	<b>C<sub>L</sub></b>	↓	↓	↑	↓	↑
		↑	↑	↓	↑	↑
6.	<b>P</b>	↓	↓	↓	↓	↓
		↑	↑	↑	↑	↓
7.	<b>γ</b>	↓	↑	*	↑	↓
		↑	↓	*	↓	↓
8.	<b>θ</b>	↓	*	*	*	*
		↑	*	*	*	*

**Note-** The arrow ↑ shows the increment and the arrow ↓ shows the decrement and \* shows constant.

## 7. Conclusion

In this paper, a model on inventory for perishable items has been explored. This paper is developed by supposing selling price dependent hybridized demand, advance payment facility, preservation technology, and partial backlogging with a fix backlogging rate. The whole study carried out under the effect of inflation. Then the compatible optimization problem related to this model is obtained as a non-linear maximization problem which is solved by applying the gradient-based technology. The convexity of the objective function formulated optimization problem is found out mathematically by the principal minor technology of the Hessian matrix.

Now from numerical analysis it concluded that the model with preservation technology provide better result than the model without preservation technology. Overall result we can say that this model can be applied in the food grains and seasonal goods like vegetables.

In the future, the inventory model can be extended by incorporating the variable demand dependent on time, freshness of the items, credit period, and a quantity discount. On the top, the model can also be further developing by inclusive multilevel trade credit policies.

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