Overview of Queueing Systems Including Mathematical Models and Applications

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ABSTRACT

One or more servers, each responsible for delivering a certain service to waiting clients, make up a queueing system. Inevitably, almost everyone has had to wait in a long line for something at some point. One can rationally assume that whomever is first in line should be given priority. However, there are times when this generalization fails to hold true. It's not uncommon for the person at the back of the line or the client with the highest priority to be served before the person who has been waiting the longest. In queueing theory, all of these features provide for fascinating study topics. Some of the papers by other researchers are shown below, along with brief summaries of their findings. We then apply some of the mathematical equations that characterise the various waiting habits. In nearly all cases, mathematical simulations are used to investigate queueing behaviour in the literature. It is a queuing model with a single server, an exponentially distributed distribution of slots, and a Poisson distribution of arrivals. In other words, it is a system with a single channel, a Poisson product, and an exponential waiting time. Queue definitely plays a major role in the scenario we are in. In the buffer, messages are queued for transmission. The Monte Carlo fashion is useful for assaying staying time problem problems which are delicate or insolvable to be anatomized mathematically dissembled slice styles are relatively helpful when FCFS isn't valid for particular queuing problem.

1. INTRODUCTION

The modern era is the time of technology and advancement. Machines are doing most of the work of humans with ease. With the growing industrial sectors and enhanced requirement of humans it becomes mandatory to add machines in daily routine work. It has made the life more comfortable and peoples are able to do their job faster with the support of machines and latest technology. Now, it became mandatory for all sectors to take the advantages of machine services to compete in the market. Every moment we spend waiting for some service, we are part of the queue. We queue in our cars in traffic jams or at tool stalls, we wait in line at supermarkets to check out, we stand in line at a hairdresser or beauty salon, we wait in line at a post office, etc. , as customers, we generally do not like such waits, and the managers of the institutions where we wait, also do not like that we wait, because then it can pay. So why wait? The answer is that the service is available. In this article, we present queuing models and their associated definitions. We define a machine perturbation problem, which is a finite source queuing model.

Queueing theory deals with the study of waiting time for a person standing in a queue to get the service. There are various types of Queueing methods exist in practice. The matter enables finding an appropriate balance between the cost of the service and the amount of waiting. Queueing models characterizes the situations where certain where certain units (arrival) known as customers, arrive in continuous time in order to receive a service or facility provided by other units called servers, when the customers arrive they immediately start to be served, if any server available, otherwise the customer must wait for the service.

Queueing theory is a methodology of a Stochastic Process. Takai, H. [1953] studied the Queueing Analysis technique. He studied the fundamentals of evaluation of finite systems. Gross D. and Harris C.M. (1985) propounded fundamentals of Queueing theory. J. Medhi (1991) discussed the methods used in Queueing models. Finite sources Queueing system and their application were studied by Sztrik in (2002). Kendall (1953) developed Queueing theory representation. With the development of the latest technology, the mechanization system has penetrated into every corner of our life. This ensures near dependency on the processing system. Over time, a machine may fail due to wear or unexpected failure and thus require corrective action, providing an opportunity for repair, after which the machine can be properly restarted. If the broken machines ever need the attention of a repair shop, the unavailability of unused repair shops can create a line of broken machines. This phenomenon requires the attention of mathematicians, because such Chinese correction models have fascinated many well-known scientists working on sequence theory.

Machine inferences occur when a single operator operates two or more machine simultaneously. The time that is spent in the call for the service and start of the service is called interference time. Several machine systems can be seen in our general lives. Our maximum work is dependent on machines but sometimes it fails to serve due to some unpredicted faults and requires maintenance. After repair work it starts serving normally. Machine breakdown can be treated as arrival of customers and repair (or replacement) of faulty spares can be treated as the service facility in the Queueing system. Machine interference model is finite source Queueing model. Many researchers have worked in the field of machine repair system. K.H. Wang, B.D. Shivazlin (1992) studied cost analysis of the model M/M/R machine repair problem with servers operating under variable service rate, J.C. Ke, S.I. Lee and C.H. Liou (2009) studied machine repair problem with balking reneging and stand by switching failure. The operating machines are outside the Queueing system and enter the system only when they breakdown and require repair shown in the following figure operating machines servers.

2. REVIEW OF LITERATURE

R. Sivaraman et. al (2017) considered a queuing model for a machine repair system following a Bernoulli holiday schedule. It assumes that failure times, repair times and vacation times follow an exponential distribution. In case of congestion, the server may increa-+se or decrease the queue length. He modeled a finite state Markov chain and its stable distribution is obtained by a matrix recursive approach.

M. Jain et.al (2009) investigated the Machine Repair Problem (MRP) and highlighted the historical development of queuing models for their practical importance. He tried to create some basic real-life MRP models of his in traffic situations. He has made several important contributions to the field of editing systems.

K.H. Wang, W.L. Cheng, and D.Y. Yang (2009) dealt with the optimal management of machine repair problems associated with company holidays and used Newton's method.

Kharram, E(2008) analyzed an optional queuing model with a dynamic number of craftsmen in a finite population queuing system. J. C.Ke and K.H. Wang (2007) gave an idea of leave policy for the problem of machine repair with two types of spare parts.

Lami, Haque, and his HJ Armstrong (2007) reviewed research on machine interference. (2006) S.P. Chen uses a mathematical programming approach to mechanical interference problems with fuzzy his parameters. Using spare parts and regression,

Madhu jain (2004) studied his N-approach for machine control systems and individually he used Alpha A.S. (2003) Create a model of holidays. S.R. Chakraborty S. (2003) introduced phase-type repair and service and examined the problem of repairing machines with unreliable servers. MJ Armstrong (2002) discussed a lifetime repair policy for mechanical repair problems. Queue length distribution for non-volatile computers was developed by P.Patrick Wong in 2000. AI. Soki worked at M/C/KN in 2000 on bulking, extraction and past nested prototypes of the machine. Budi Hardigo, I. and David, H.T. (1999) presented a ranking of expected wait times for extended machine repair models. Gupta, S.M. (1997) solved the problem of mechanical interference with warm spare parts. CH Ng (1997) discussed the foundations of queuing theory. Jalalov J.H. (1997) developed a queuing system with state-dependent parameters. A.N. Shirayaen, A.N. Shiriaer (1996) discussed his book titled Probability by his R.P. Boas, and E.E. Lewis (1196) gave an overview of reliability engineering.

Fluency Standard R. Sivaraman et. al (2017) considered a queuing model for a machine repair system following a Bernoulli holiday schedule. It assumes that failure times, repair times and vacation times follow an exponential distribution. In case of congestion, the server may increase or decrease the queue length. He modeled a finite state Markov chain and its stable distribution is obtained by a matrix recursive approach. C.

M. Jain et.al (2009) surveyed machine repair problems (MRP) and emphasized on historical developments of queuing models for the practical importance. He attempted to elaborate some basic MRP models of the real life during traffic situations. He provided some important contributions done in the area of machining system.

Sushil Ghimire et.al (2017) proposed the use of finite capacity queueing models in which the lesser number of customers were served by a single or numerous numbers of servers and the batch queueing model for the study where coming or getting service or both happens in a bulk. A. Abate and W. Whitt (1987) had done useful research in this area. Some other studies and research are done by Feller W. (1967) , K.S. Trivedi (1982) , Elsared, E.A. (1981), D.L. Smith (1978).

3. 1 M/M/1 QUEUEING TECHNIUE

It is a queuing model with a single server, an exponentially distributed distribution of slots, and a Poisson distribution of arrivals. In other words, it is a system with a single channel, a Poisson product, and an exponential waiting time. In First in first out mode, the system has infinite capacity for queues. The first M represents the Poisson input, the second M the Poisson output, 1, the number of servers, and indicating the unlimited capacity of the system.

3.1 Behavior of Steady State

In the single server system, input system pattern Poisson and service time follows exponential distribution with First Come first Serve queuing system, known as simple queue, may be seen.

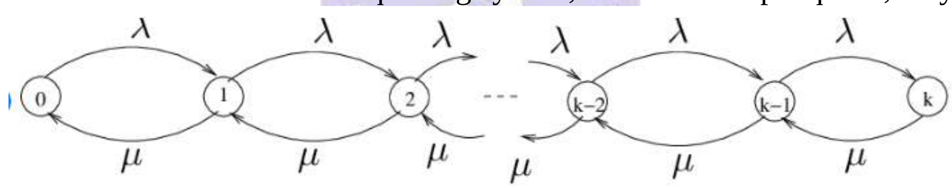

Figure 1: Transition Graph for M/M/1 Queue

$$-\lambda P_0 + \mu P_1 = 0 \quad (1)$$

$$-(\lambda+\mu)P_n + \lambda P_{n-1} + \mu P_{n+1} = 0, \; n = 1, 2, 3, \ldots \quad (2)$$

$$\lambda P_0 = \mu P_1 \quad (3)$$

We get using,

$$P_n = P^0 \rho^n; \; n \ge 0 \quad (4)$$

$$\sum_{n=0}^{\infty} P_n = 1 \quad (5)$$

$$P_n = (1-\rho)\rho^n; \quad n \ge 0 \quad (6)$$

3.2 Limited Waiting Space Queue (M/M/1/K) Model

In this model maximum number of K units (including that one being served)

$$\lambda P_0 = \mu P_1 \quad (7)$$

$$(\lambda+\mu)P_n=\lambda P_{n-1}+\mu P_{n+1} \quad 1\le n\le K-1 \tag{8}$$

$$\mu P_k \;=\; \lambda P_{k-1} \tag{9}$$

Simplifying

$$P_n \;= P_0\rho^n \qquad 0\le n\le K-1 \tag{10}$$

Using

$$\sum_{n=0}^{k} P_n \;=1$$

$$\therefore P_0 \;=\frac{1-\rho}{1-\rho^{k+1}}; \quad \rho\ne 1 \tag{11}$$

$$=\frac{1}{k+1}; \qquad \rho=1 \tag{12}$$

For n=0.1.2.3…..k.

$$\rho_n=\frac{(1-\rho)\rho^n}{(1-\rho)^{k+1}}; \;\rho\ne 1 \tag{13}$$

$$L_s=\sum_{n=1}^{k} nP_n \tag{14}$$

The distribution is truncated geometric when $\rho\ne 1$ and uniform when $\rho=1$

Mean Number $\rho=1, L_k=\dfrac{K}{2}$ (15)

$$\rho=1, L_k=\frac{K}{2} \tag{16}$$

For $For\rho\ne 1,$

$$L_k=\frac{(1-\rho)\rho}{1-\rho^{k+1}}*\frac{(1-(k+1)\rho^k+x\rho^{k+1}}{(1-\rho)^2} \tag{17}$$

Now we consider Erlang Model.

3.3 THE $M/E_K/1$ QUEUEING MODEL

A basic example in the model in the Erlang K service time distribution

Average service rate

$$E_k(x)=1-\sum_j\frac{x}{j!}\mu^j e^{-\mu x} \tag{18}$$

For large K.

4. COMMUNICATION BUFFERS

Suppose we have a focus node that accepts messages from multiple sources and sends them through an aggregated output channel. Queue definitely plays a major role in the scenario we are in. In the buffer, messages are queued for transmission.

In many cases where we have fixed-length messages, M/D/1model of queue is applicable. Message if transmitted in time μ then required service time is

$$\lambda = \frac{1}{\mu} \qquad (19)$$

Mean number in the system

$$N = \frac{\rho}{1-\rho}(1-\rho/2) \qquad (20)$$

Mean time in system

$$W = \frac{1}{\mu-\lambda}(1-\rho/2) \qquad (21)$$

Example 1

For 8 hours a day, a user's tolerance a loss of more than 10 messages in a multiplexer. To provide this level of service, how many buffers are required?

Solution:

There are

$$100*3600*8 = 288*10^4 \text{ messages in a day}$$

Of these no more than 10 messages should be lost. Prospect

$$\frac{10}{288*10^4} = 3.5*10^{-6} \qquad (22)$$

Don't exceed in all buffers

Prob. That there are n-customers in the system is ρ^{n+1}

$$\rho^{n+1} < 3.5*10^{-6} \qquad (23)$$

$$or, (n+1)\log_e \rho < -12.57 \qquad (24)$$

$$\Rightarrow n > -12.57/\log_e \rho - 1 \qquad (25)$$

5. APPLICATION OF MONTE CARLO TECHNIQUE IN QUEUEING PROBLEM

Monte Carlo simulation is a mathematical technique that calculates the probabilities of several possible outcomes of an uncertain process using repeated random sampling. This calculation algorithm facilitates the assessment of risks associated with a certain process and enables better decision-making Monte Carlo simulation is a fashion frequently used in threat operation, but numerous interpreters don't understand its significance. utmost of the PMP applicants find this conception veritably delicate to understand. thus, they believe that computer software should be used to perform Monte Carlo simulations. This system is frequently used by professionals in numerous fields similar as finance, engineering, energy, design operation, manufacturing, exploration and development, insurance, transportation and terrain to break numerous problems. The Monte Carlo fashion is useful for assaying staying time problem problems which are delicate or insolvable to be anatomized mathematically dissembled slice styles are relatively helpful when FCFS isn't valid for particular queuing problem. In numerous cases, the observed distribution for appearance times and service times can not be filled to certain Mathematical distribution and Monte Carlo approaches the only stopgap under similar situations. Multi-channel queuing problem can be fluently handled by this fashion. The Monte Carlo approach has numerous advantages over the ordinary slice system of just looking at the factual situation and husbandry history of advents, services, line lengths and staying times.

6. CONCLUSIONS

The modern era is the time of technology and advancement. Machines are doing most of the work of humans with ease. With the growing industrial sectors and enhanced requirement of humans it becomes mandatory to add machines in daily routine work. It has made the life more comfortable and peoples are able to do their job faster with the support of machines and latest technology. Now, it became mandatory for all sectors to take the advantages of machine services to compete in the market. Every moment we spend waiting for some service, we are part of the queue. We queue in our cars in traffic jams or at tool stalls, we wait in line at supermarkets to check out, we stand in line at a hairdresser or beauty salon, we wait in line at a post office, etc. as customers, we generally do not like such waits, and the managers of the institutions where we wait, also do not like that we wait, because then it can pay. So why wait? The answer is that the service is available. In this article, we present queuing models and their associated definitions. We define a machine perturbation problem, which is a finite source queuing model. The present article investigated and defines queuing models and related topics of the models. We have defined some basic definitions of queues and distributions. At the end of this article, we discussed simulation, its benefits and drawbacks as well as available software packages, Research in the fields of queuing theory and machine interference issue have done some ground-breaking work, which we've covered. The present article analyzed some queuing problem methods and some examples. We have discussed the various queuing models .We have also discussed communication buffers and finally we have point out some particulars of Monte Carlo technique

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