

Mathematical Modelling of Garbage Mixing and Dispersion in the Damodar River Under Varying Flow Conditions

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Abstract

The Damodar River is an important river system of eastern India and passes through regions characterized by intensive industrial, mining, agricultural and urban activities. These activities generate different forms of solid waste and other pollutants that may enter the river directly or indirectly through surface runoff, drainage channels, municipal discharge and local dumping. The movement and mixing of garbage within a river are strongly influenced by hydraulic conditions, particularly flow velocity, discharge, channel geometry, turbulence and longitudinal dispersion. The present study proposes a mathematical framework for analysing the transport, mixing and dispersion of garbage-associated pollutant loads in the Damodar River under varying flow conditions. A one-dimensional advection–dispersion model is proposed in which the longitudinal transport of a representative garbage-derived pollutant concentration is expressed as a function of flow velocity and dispersion coefficient. The model incorporates garbage input, transport, dispersion and first-order removal processes. Flow-dependent dispersion is represented through a relationship between the dispersion coefficient and mean river velocity. The model can be calibrated using observed river discharge, velocity, channel dimensions and pollutant concentration at selected monitoring locations. Different flow scenarios, including low-flow, normal-flow and high-flow conditions, can then be simulated to determine changes in pollutant concentration and downstream dispersion distance. The proposed framework provides a quantitative approach for understanding the relationship between river flow and garbage-related pollution transport. It can support river monitoring, pollution-load estimation and environmentally informed waste-management planning for the Damodar River system.

Keywords: Damodar River, mathematical modeling, garbage mixing, solid waste, advection, dispersion, river flow, pollutant transport, environmental modeling.

Introduction

Rivers are dynamic systems in which pollutants introduced from human settlements, industries, agricultural areas and drainage networks are transported and transformed under changing hydraulic conditions. The movement of pollutants in a river does not occur independently of river flow. Discharge, flow velocity, channel depth, width, turbulence and longitudinal mixing determine how rapidly a pollutant moves downstream and how its concentration changes with distance.

The Damodar River represents an important case for studying pollution transport because its basin contains industrial, mining, agricultural and densely populated areas. Previous research has documented substantial spatial and seasonal variation in water-quality characteristics within the Damodar River system. Studies have also examined the influence of land use, runoff, discharge and other hydro geomorphological factors on river pollution. Therefore, the river provides an appropriate setting for developing a flow-dependent mathematical framework for pollutant transport.

Garbage entering a river may occur in different physical forms. Large floating materials may be transported near the surface, while smaller organic and suspended waste particles can become incorporated into the water column. Organic matter associated with waste can also generate measurable pollutant concentrations such as biochemical oxygen demand (BOD), chemical oxygen demand (COD), total suspended solids (TSS) and other water-quality indicators. Consequently, instead of treating every individual piece of garbage as a separate mathematical particle, the present study represents garbage pollution through an equivalent pollutant concentration or pollutant load.

The central assumption of this study is that the downstream distribution of garbage-associated pollution can be represented through the combined effects of advection, dispersion and removal. Advection represents movement with the mean river flow, whereas dispersion represents the spreading and mixing of the pollutant along the river. A mathematical model

incorporating these processes can help explain why pollution concentration and distribution may differ considerably between low-flow and high-flow periods.

Recent research on the Damodar basin has demonstrated that discharge, runoff, groundwater flow, land use and seasonal conditions influence river-water quality. Such findings support the need for models that explicitly incorporate hydrological variability rather than considering pollution concentration independently of river flow.

Research Problem

The uncontrolled disposal and transport of solid waste into river systems can create environmental problems downstream. However, the impact of waste entering a river is not determined only by the quantity of waste introduced. Its downstream behavior also depends upon hydraulic conditions.

During periods of low flow, the dilution capacity of the river may decrease, potentially resulting in relatively high concentrations of pollution. During periods of high flow, the increased transport capacity can move waste and associated pollutants over longer distances. At the same time, turbulence and mixing may increase dispersion. Thus, high flow may simultaneously increase downstream transport and dilution.

A major research problem is therefore to establish a mathematical relationship between river flow and the longitudinal distribution of garbage-associated pollution.

The present study addresses this problem through a mathematical advection–dispersion framework.

objectives of the Study

The major objectives are:

1. To develop a mathematical model for garbage-associated pollutant transport in the Damodar River.
2. To examine the influence of river flow velocity on pollutant transport.
3. To estimate the longitudinal dispersion of garbage-associated pollution under different flow conditions.

Hypotheses

H₀₁: River-flow velocity has no significant effect on the longitudinal transport of garbage-associated pollutants in the Damodar River.

H₁₁: River-flow velocity has a significant effect on the longitudinal transport of garbage-associated pollutants in the Damodar River.

H₀₂: There is no significant relationship between river discharge and downstream pollutant concentration.

H₁₂: There is a significant relationship between river discharge and downstream pollutant concentration.

Study Area

The study focuses on the Damodar River system in eastern India. The Damodar is a major tributary of the Ganga and flows through Jharkhand and West Bengal. A substantial part of its basin lies in Jharkhand, while downstream reaches extend through West Bengal.

The river basin contains industrial and mining regions as well as agricultural and urban areas. These land-use characteristics create multiple potential pathways through which pollutants and waste materials may enter the river.

For mathematical modeling, a selected river reach should be identified on the basis of:

- availability of discharge data;
- availability of flow-velocity measurements;
- presence of identifiable waste-entry points;
- accessibility of sampling locations;
- relatively consistent channel geometry;
- availability of upstream and downstream water-quality observations.

The selected reach may be divided into monitoring stations such as:

$$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_5$$

where S_1 represents the upstream reference location and S_2 – S_5 represent progressively downstream locations.

Conceptual Framework

The proposed garbage-transport system can be represented as:

**Garbage Input → River Entry → Advection → Mixing → Dispersion →
 Transformation/Removal → Downstream Transport**

The main components are:

Garbage Input

Garbage enters the river through:

- municipal drainage;
- surface runoff;
- direct dumping;
- settlement areas;
- industrial areas;
- agricultural runoff;
- tributary channels.

Advection

Advection represents downstream movement of pollutants due to the mean velocity of river water.

Dispersion

Dispersion represents spreading caused by velocity variations, turbulence and mixing.

Transformation

Some organic components of garbage may undergo decomposition, while suspended materials may settle or become resuspended.

Removal

Pollutants may be removed from the water column through:

- sedimentation;
- biological degradation;
- adsorption;
- bank deposition;
- extraction or cleanup activities.

Mathematical Model

Basic Advection–Dispersion Equation

The one-dimensional advection–dispersion–reaction equation is proposed as the basic model:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2} - kC + S$$

Where:

- C = concentration of garbage-associated pollutant
- t = time
- x = longitudinal distance along the river
- u = mean river-flow velocity
- D = longitudinal dispersion coefficient
- k = first-order decay/removal coefficient
 - S = pollutant source/input term

This equation represents the balance among transport, dispersion, removal and external input.

Flow-Dependent Dispersion Model

The dispersion coefficient can be represented as a function of river velocity.

A simple relationship is:

$$D = \alpha u$$

Where:

- D = longitudinal dispersion coefficient;
- u = mean flow velocity;
- α = empirical dispersion parameter.

$$D = \alpha u^m$$

where m is an empirically estimated exponent.

The value of α and m should be estimated from observed field data rather than assumed arbitrarily.

9. Flow and Velocity Relationship

River discharge is given by:

$$Q = Au$$

Where:

- Q = river discharge;
- A = cross-sectional area;
- u = mean flow velocity.

The cross-sectional area can be approximated as:

$$A = B \times H$$

where:

- B = average channel width;
- H = average water depth.

Therefore:

$$u = \frac{Q}{BH}$$

This relationship provides a direct connection between discharge and pollutant transport.

Steady-State Simplified Model

If the system is considered approximately steady and there is no continuous distributed source along the selected river reach, the equation becomes:

$$u \frac{dC}{dx} = D \frac{d^2C}{dx^2} - kC$$

This equation can be used to estimate how pollutant concentration changes with downstream distance. For a simplified case where longitudinal dispersion is relatively small compared with

$$u \frac{dC}{dx} = -kC$$

advection:

The solution is:

$$C(x) = C_0 e^{-\frac{kx}{u}}$$

where:

- C_0 = initial concentration at the pollution-entry point;
- $C(x)$ = concentration at downstream distance x ;
- k = removal coefficient;
- u = mean flow velocity.

This equation shows how the predicted concentration profile changes when river velocity changes.

Incorporating Garbage Load

Garbage can be represented through a mass-loading term.

The pollutant load entering the river can be expressed as:

$$L = M_g f_p$$

where:

- L = pollutant mass load;
- M_g = garbage mass entering the river;
- f_p = fraction of garbage contributing to the selected pollutant parameter.

If the river discharge is Q , the corresponding initial concentration can be estimated as:

$$C_0 = \frac{L}{Q}$$

after applying appropriate unit conversions.

This relationship is important because the same garbage load may produce different concentrations under different discharge conditions.

For example, if the garbage-associated pollutant load remains constant but river discharge increases, the initial concentration may decrease because of increased dilution.

Proposed Results Presentation

Since actual field observations are required for numerical results, the following structure is recommended for the final empirical analysis.

Table 1. Hydraulic Characteristics

Flow Condition	Discharge Q (m ³ /s)	Velocity u (m/s)	Width B (m)	Depth H (m)
Low Flow	Actual data	Actual data	Actual data	Actual data
Normal Flow	Actual data	Actual data	Actual data	Actual data
High Flow	Actual data	Actual data	Actual data	Actual data

Table 2. Garbage/Pollutant Loading

Flow Condition	Garbage Load (kg/day)	Initial Concentration	BOD	COD	TSS
Low Flow	Field data	Field data	Field data	Field data	Field data
Normal Flow	Field data	Field data	Field data	Field data	Field data
High Flow	Field data	Field data	Field data	Field data	Field data

Table 3. Observed and Predicted Concentration

Station	Distance (km)	Observed C	Predicted C	Error
S ₁	0	Field data	Model output	Calculated
S ₂	Actual	Field data	Model output	Calculated
S ₃	Actual	Field data	Model output	Calculated
S ₄	Actual	Field data	Model output	Calculated
S ₅	Actual	Field data	Model output	Calculated

Discussion

The mathematical framework developed in this study emphasizes the importance of considering river hydraulics while analysing garbage-related pollution. A simple measurement of waste quantity does not provide sufficient information about its environmental distribution. Two periods with similar garbage input may produce different downstream concentration profiles because discharge and flow velocity can differ considerably.

Under low-flow conditions, the river's dilution capacity may be comparatively limited. Consequently, pollution concentrations can remain relatively high within the affected reach. Under high-flow conditions, greater discharge can enhance dilution, while increased velocity can simultaneously transport pollutants more rapidly downstream.

This dual effect is particularly important for river management. A reduction in concentration at one monitoring station during high flow should not automatically be interpreted as a reduction in the total pollution burden. The pollutant may instead have been transported farther downstream.

The model also provides a basis for distinguishing between concentration and pollutant load. Concentration represents the amount of pollutant per unit volume of water, whereas load represents the total mass transported per unit time. Therefore, both parameters should be examined when evaluating the environmental significance of garbage transport.

Previous Damodar River research has similarly shown that discharge, runoff, land use and seasonal conditions influence water-quality behavior. A recent basin-scale study found that industrial/urban areas, runoff, groundwater flow and agricultural areas can have different effects during different seasons.

Future Scope

Future studies can extend the proposed model from one-dimensional transport to two-dimensional or three-dimensional hydrodynamic simulations. Floating, suspended and deposited waste can be represented as separate compartments. Remote sensing and GIS can be incorporated to identify garbage accumulation zones and waste-entry locations. A coupled hydrodynamic–water-quality model could also integrate discharge, sediment transport, BOD, COD, nutrients, heavy metals and solid-waste movement. Machine-learning techniques may subsequently be used with the mathematical model to improve short-term prediction. Future research may also incorporate rainfall-runoff relationships so that sudden increases in waste transport following rainfall events can be simulated. Integration with real-time river monitoring sensors could eventually permit near-real-time prediction of pollutant transport.

Conclusion

The present study proposes a mathematical framework for examining garbage mixing and dispersion in the Damodar River under varying flow conditions. The central model is based on the advection–dispersion–reaction equation, which incorporates river velocity, longitudinal dispersion, pollutant loading and removal processes.

The model establishes an important conceptual relationship between river discharge and garbage-associated pollution. Flow velocity determines the rate at which pollutants are transported, while dispersion controls their spatial spreading. Discharge also influences dilution and therefore affects concentration. Consequently, pollution concentration, pollution load and downstream transport distance should be analysed together.

The proposed framework can be calibrated with field observations and used to simulate low-flow, normal-flow and high-flow conditions. Its principal contribution is the integration of river hydraulics with garbage-associated pollutant transport in a mathematical framework. With appropriate field data, the model can be developed into a site-specific predictive tool for monitoring, assessment and management of pollution in the Damodar River.

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